



Banc Ceannais na hÉireann
Central Bank of Ireland

Eurosystem

Research Technical Paper

Estimating the Trend of House Price to Income in Ireland

Fang Yao

Vol. 2022, No. 8

Estimating the Trend of the House Price to Income Ratio in Ireland

Fang Yao^{*}

November 24, 2022

Abstract

Distinguishing trends and cycles in house prices is important for macroprudential policy. This paper estimates the unobserved trend and cycles of the house-price-to-income ratio (HPI) using a multivariate unobserved components model, with auxiliary variables introduced for identifying both the trend and cycles. Under this approach, the HPI trend is driven by slow-moving fundamental forces, while the cyclical component of HPI is identified by using separate cyclical indicators in a VAR. I find that the estimated trend of the HPI ratio is rising over time, driven primarily by the declining natural interest rate, and to a lesser extent by credit conditions and housing imbalances. Of relevance for macroprudential policy setting, the underlying trend in HPI has risen by 8% since the introduction of borrower-based measures in 2015.

JEL classification: E5, G01, G17, G28, R39.

Keywords: real estate markets, macroprudential policy, systemic risk, financial crises, bubbles, financial regulation, financial stability indicators.

^{*}I thank Robert Kelly, Gerard Kennedy, Reamonn Lydon, Paul Lyons, Vasileios Madouros, Fergal McCann, Niall McGeever, Martin O'Brien, Michael O'Grady, Sofia Velasco and other seminar participants for helpful comments and suggestions. The views express in this paper are those of the author and do not necessarily represent the views of the Central Bank of Ireland. email fang.yao@centralbank.ie.

Non-technical summary

House price movements can be decomposed into slow-moving underlying trends and cyclical forces. Distinguishing between these two forces is important for financial stability assessments and macroprudential policy.

This paper contributes to the existing literature on estimating the unobserved trend and cyclical components of the house price to income ratio in Ireland. I enrich the multivariate unobserved components models by introducing both auxiliary variables for trend and cyclical components. The advantage of this approach is two-fold. First, the estimated trend is explicitly modeled as being driven by long-run fundamental factors. This approach makes the estimated trend economically interpretable and therefore increases the usefulness for policy purposes. Second, introducing cyclical indicators into the model help the estimated cyclical component to be consistent with historical experiences of financial cycles.

Estimation of trend and cyclical factors can be an useful input to deliberations over the calibration of macroprudential policy instruments such as the Loan to Income ratio. Where long-run, structural changes in the equilibrium HPI are identified in a methodology such as this, they can be used as motivating factors in analysing the merits of recalibration of borrower-based measures. Since the implementation of macroprudential mortgage measures in 2015 in Ireland until the finalisation of the Central Bank's first framework review in October 2022, the HPI trend has risen by 8% from 3.8 in 2015 to 4.1 by the end of 2021. This increase in trend HPI suggests that a flat LTI calibration has been slowly become more binding in Ireland since 2015, even before accounting for the impact of cyclical forces on access to the mortgage market. The main driving force is the low natural interest rate, which has fallen from around 8% in the early 1990's to -2% most recently.

1 Introduction

Housing markets played a key role in sowing the seeds of the Global Financial Crisis (GFC) and in propagating the European sovereign debt crisis. As a result, monitoring and assessing housing market imbalances has become standard practice for financial stability surveillance. The key challenge faced by economists is to distinguish cyclical forces in house prices from the trend growth driven by long-run supply and demand fundamentals, as not all house price increases are necessarily evidence of cyclical imbalances.

This paper proposes a novel method to estimate the trend and cycle component of the house price to income ratio (HPI hereafter)¹ with multivariate state-space models. The main contribution of the paper is that I enrich the multivariate unobserved components models in the existing literature by introducing auxiliary variables for both unobserved trend and cyclical components. The advantage of this approach is two-fold. First, the estimated trend is not identified as a random walk process, instead, it is explicitly modeled as being driven by long-run fundamental factors. This approach makes the estimated trend economically interpretable and therefore increases the usefulness for policy purposes. Second, introducing cyclical indicators into the model help the estimated cyclical component to be consistent with historical experiences of financial cycles.

More specifically, under this approach, the HPI trend is explicitly modeled as being driven by slow-moving housing supply/demand imbalance, the natural interest rate and credit conditions, while the cyclical component is identified by using cyclical variables, such as the mortgage growth rate and the unemployment rate, in a unrestricted Vector Autoregression (VAR) model. These modeling specifications are then fit into the state-space and estimated by the maximum likelihood estimator with quarterly time series data from Ireland between 1984 Q1 and 2021 Q4.

I develop a number of empirical findings: First, the estimated trend of the HPI ratio in Ireland is rising over time, driven primarily driven by the falling natural interest rate and, to a lesser extent, by other factors such as credit conditions and the housing supply/demand imbalances. Variance decomposition shows that around 80% of variance of the HPI trend can be attributed to the natural interest rate, which has fallen from around 8% in the early 1990's to -2% most recently.

Second, the estimated VAR model of financial cycles shows highly persistent dynamics, with diagonal coefficients of all cyclical variables being highly significant and close to one. On the off-diagonal coefficients, I find that the unemployment rate has a significant negative correlation with the cyclical component of HPI, while mortgage growth shows a significant positive correlation. More importantly, the estimated cycles show boom-bust cycles of housing markets that are consistent with the historical experiences in Ireland.

This paper intersects the literature in housing and financial stability. In this literature, the most commonly used methods to monitor the housing gap generally fall into one of two categories. First, long-run averages of the price-to-income ratio or the price-to-rent ratio are used to assess whether house prices have significantly deviated from their

¹ I choose to focus on HPI, instead of the house price itself as many studies in the literature, because HPI is a better measure of housing market vulnerability, while the house price growth might reflects other factors, such as economic growth outside housing markets. In addition, the loan to income ratio is one of widely used macroprudential tools, and estimated changes in the trend of HPI help to inform calibration of LTI more directly.

fundamentals (See: e.g. [Girouard et al., 2006](#); [Kennedy et al., 2016](#); [Philipponnet and Turrini, 2017](#)). Under this approach, the trend is assumed to be constant over time. This is at odds with the idea that trends are driven by long-run fundamentals, which are known to vary over time. In addition, the price-to-rent ratio compares the cost of owning and renting a house with the underlying assumption that there is a high degree of arbitrage between the rental and ownership markets.² The second category follows the housing literature, and directly models fundamental house prices by estimating a cointegrating relationship between house prices and a number of macroeconomic variables that reflect the demand and supply fundamentals of housing markets.³ Multivariate cointegration analysis mainly focuses on the trend component, while residuals from the linear regression are interpreted as housing cycles. This approach renders an unsatisfactory estimate of housing cycles, as by assumption, residuals from a linear regression should not be serial correlated, which is at odds with the fact that housing cycles might be highly persistent. In my model, by contrast, housing cycles are allowed to interact with other cyclical variables in a VAR setting.

The methodology used in this paper has roots in the trend-cycle decomposition literature using unobserved components (UC) models.⁴ UC models are widely used to decompose business and financial cycles.⁵ Using a univariate UC model, [Lucas and Koopman \(2005\)](#) and [Galati et al. \(2016\)](#) extract cycles from unobserved trend with the US and Euro area data. The underlying assumption is that the trend follows a random walk process and cyclical components follow an AR(2) process. [O'Brien and Velasco \(2020\)](#) decompose the credit gap with a multivariate UC trend-cycle model with stochastic volatility. Their trend component is also based on the random walk assumption. The random walk assumption implies that the trend component is expected to be explosive over time, which is hard to justify theoretically, especially for variables such as house-price-to-income ratio. Furthermore, the random-walk assumption also rules out the possibility that the trend can be explained by structural fundamental drivers of housing markets. My approach, by contrast, doesn't rely on a random walk assumption to identify the trend of HPI; instead, I allow the trend component to be explicitly driven by fundamental factors and estimated directly from the data. Similar to my approach, [Lang and Welz \(2018\)](#) and [Galán and Mencía \(2018\)](#) study credit ratios and explicitly model the trend component of credit as being driven by fundamental economic factors. The cyclical component in their models, however, is assumed to follow an AR(2) process.⁶

² This assumption is not applicable to many OECD countries, including Ireland, where regulated rental markets and restrictions on land uses for residential houses or zoning regulation widely present.

³ See: e.g. [Abraham and Hendershott \(1994\)](#), [Muellbauer and Murphy \(1997\)](#). [Geng \(2018\)](#) studies the long-run determinants of house prices in a 20-country panel to estimate the degrees of over-valuation for different countries.

⁴ Another class of the decomposition technique is non-parametric filters, such as [Hodrick and Prescott \(1997\)](#), [Baxter and King \(1999\)](#), [Christiano and Fitzgerald \(1999\)](#). [Lowe and Borio \(2002\)](#) use Hodrick and Prescott filter to decompose credit-to-GDP ratios. [Aikman et al. \(2017\)](#) apply a Christiano and Fitzgerald filter to identify financial cycles. A recent summary can be seen in [Hodrick \(2020\)](#) and associated criticisms by [Hamilton \(2018\)](#)

⁵ For the applications in business cycle literature, see, e.g. [Beveridge and Nelson \(1981\)](#), [Nelson and Plosser \(1982\)](#), [Clark \(1987\)](#), [Harvey \(1991\)](#), [Morley et al. \(2003\)](#) among others.

⁶ [Galán and Mencía \(2018\)](#) also allows the credit cycle to interact with house prices in a linear relationship.

The rest of the paper is organized as follows. In section 2, I describe the empirical specification and data. Section 3 presents empirical results from the benchmark model. In Section 4, I consider robustness of the baseline model and conduct a series of robustness tests. Section 5 discusses the policy implication of the decomposition result. In Section 6, I conclude.

2 Empirical specification

2.1 House price to income ratio in Ireland

For the empirical analysis, I use a quarterly time series of the HPI ratio from Ireland between 1984:Q1 and 2021:Q4. As shown in Figure (1), the HPI ratio is volatile in Ireland over the sample period. It starts from relative low levels in the 1980s and early 1990s, when the Irish economy suffered from a persistence of economic turmoil. The HPI series started to pick up rapidly from 1995 during the ‘Celtic Tiger’ period and ended abruptly with the outbreak of GFC in 2007/08. Since then, the Irish HPI has experienced a sharp fall that lasted until the end of the European Sovereign Debt Crisis around 2013. A strong recovery in the housing market has since then pushed the HPI ratio upward, but it was interrupted by the outbreak of the COVID-19 pandemic. Overall, the HPI series in the sample captures not only two large boom-bust financial cycles that have been experienced in the recent decades, but also reflects long-run trend developments, which are driven by slow-moving demand and supply fundamentals of the Irish housing markets. The empirical exercise is intended to disentangle those long-run trend developments from the cyclical components of HPI in Ireland.

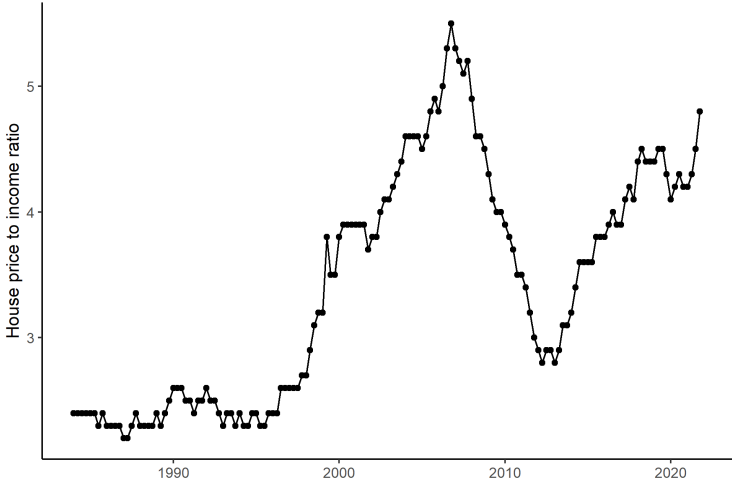


Figure 1. House price to income ratio in Ireland

2.2 Unobserved components models

The empirical modeling is based on UC models, which treat trend and cycles as unobserved components of the underlying time series of interest. This approach allows researchers to identify those unobserved components by imposing structural assumptions on the data-generating process governing the trend and cyclical components, and estimate those unknown coefficients based on the parametric assumptions and data. The main advantages

of using this approach are two-fold: first, the modeling approach is flexible, allowing a wide range of time series models to be considered in a uniformed framework; second, the modeling framework allows for the inclusion of additional variables into the analysis, to identify the unobserved trend and cycles. The general form of UC models can be expressed as follows:

$$Y_t = F_t\theta_t + v_t, \quad v_t \sim N_m(0, V_t) \quad (1)$$

$$\theta_t = G_t\theta_{t-1} + w_t, \quad w_t \sim N_p(0, W_t) \quad (2)$$

where G_t and F_t are matrices with model coefficients and v_t and w_t are two independent white noise sequences, with zero mean and covariance matrices V_t and W_t respectively. The first equation is called the observation equation, linking observable variable of interest (Y_t) to the unobservable state variables (θ_t) - in the decomposition context, they are the trend and cyclical components. The second equation is the state equation, which describes the dynamic process of states. Furthermore, it is assumed that θ_0 has a Gaussian distribution,

$$\theta_0 \sim N_p(m_0, C_0),$$

for some non-random vector m_0 and matrix C_0 .

The Kalman filter recursions are applied to obtain filtered and smoothed estimates of the unobserved components, as in Equation (1) and (2). A Maximum Likelihood estimator is used to estimate the unknown variances of the disturbances, as well as unknown coefficients in F_t and G_t .⁷

In the business cycle literature, a wide range of trend-cycle decomposition models have been proposed and studied under the state-space framework. For example, Clark (1987) assumes the trend component follows a random walk and the cyclical component is governed by an AR(2) process. Morley et al. (2003) model GDP as an ARIMA(2,1,2) and Harvey and Jaeger (1993) estimate an unobserved components model in which the trend is an ARIMA(0,2,1) and the cycle is an ARIMA(2,0,1).

2.2.1 Univariate approach

Before exploring the multivariate models, as a benchmark, I first demonstrate a trend-cycle decomposition under the univariate setting. The univariate model is based on a simple version of a Beveridge-Nelson decomposition model in which the trend is a random walk with drift and the cyclical component is a AR(1) process.

$$y_t = \tau_t + c_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2) \quad (3)$$

$$\tau_t = \tau_{t-1} + g + \mu_t, \quad \mu_t \sim N(0, \sigma_\mu^2) \quad (4)$$

$$c_t = \rho c_{t-1} + \xi_t, \quad \xi_t \sim N(0, \sigma_\xi^2) \quad (5)$$

The estimation results are shown in Table (1). The AR(1) coefficient (ρ) is highly persistent, but the drifting coefficient (g) is small and not significant from zero. The residual diagnostic plots show that residuals are autocorrelated at the third and fourth lags, indicating that the simple decomposition model fails to capture the serial autocorrelation in the HPI.

⁷ For details of this estimation method, see Schwappe (1965), and Harvey (1991). Computations are done by using MARSS library (Version '3.11.4') in R.

Table 1. Estimation Results of Beveridge-Nelson Decomposition

	estimate	std.error	conf.low	conf.up
ρ	0.9851	0.0581	0.8713	1.0989
g	0.0053	0.0024	0.0006	0.0100
σ_{μ}	0.0006	0.0021	-0.0036	0.0048
σ_{ξ}	0.0007	0.0022	-0.0035	0.0050
σ_{ϵ}	0.0000	0.0001	-0.0002	0.0002

Note: Log-likelihood: 286.7, AIC: -562.9.
 Confident intervals are calculated at 0.05 and 0.95 percentiles via hessian method.

Figure (2) shows the decomposition results. Two noticeable features of the decomposition are worth highlighting. Firstly, the estimated trend is not smooth, and evolve in a similar fashion as the cycles. Secondly, estimated cycles of HPI show a housing market boom in the 1980s and early 1990s. This is the period in which Ireland suffered one of worst economic recessions in recent history. These unsatisfactory features of the decomposition result put serious doubt on the univariate approach as a useful decomposition model for the Irish HPI.

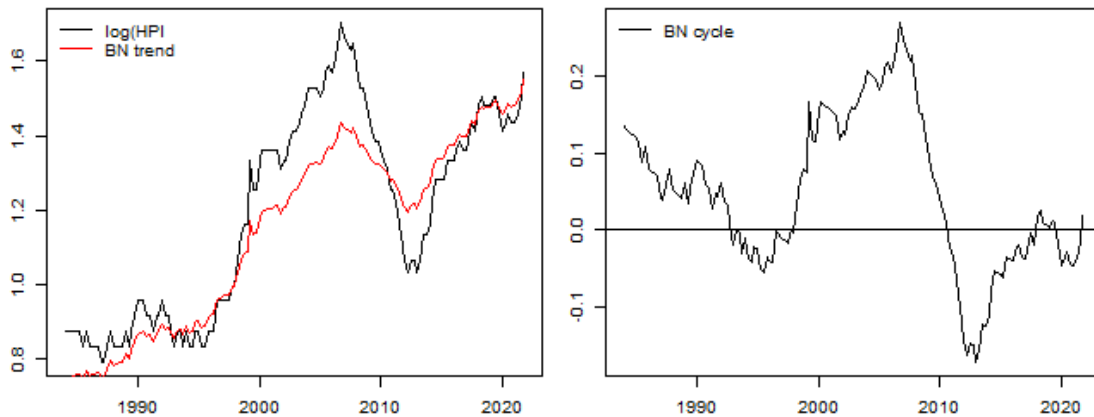


Figure 2. Beveridge-Nelson Decomposition

The univariate approach relies exclusively on parametric assumptions to identify trend and cycles of underlying data. In this model, the trend of the HPI ratio is assumed to follow a random walk process, which cannot be justified for the HPI ratio theoretically, as it implies that the trend of house prices relative to income can grow without limits over time. Next, I explore the multivariate models and see how including additional indicator variables can improve the trend-cycle decomposition.

2.2.2 Multivariate model

In the multivariate setting, I introduce a range of auxiliary variables into the UC model. Firstly, I follow Lang and Welz (2018) and Galán and Mencía (2018) to include demand and supply fundamentals of housing markets as exogenous drivers of the trend component of the HPI ratio. Secondly, I also consider a range of cyclical indicators that provide

useful information about the business and financial cycles in Ireland. The multivariate decomposition model with indicator variables takes the following form:⁸

$$y_t = \tau_t + c_t \quad (6)$$

$$\tau_t = \alpha_t^p P_t + \alpha_t^r R_t + \alpha_t^s S_t + \mu_t, \quad \mu_t \sim N(0, \sigma_\mu^2) \quad (7)$$

$$\alpha_t^p = \alpha_{t-1}^p + \zeta_{1t}, \quad \zeta_{1t} \sim N(0, \sigma_1^2) \quad (8)$$

$$\alpha_t^r = \alpha_{t-1}^r + \zeta_{2t}, \quad \zeta_{2t} \sim N(0, \sigma_2^2) \quad (9)$$

$$\alpha_t^s = \alpha_{t-1}^s + \zeta_{3t}, \quad \zeta_{3t} \sim N(0, \sigma_3^2) \quad (10)$$

$$X_t = AX_{t-1} + \Xi_t, \quad \Xi_t \sim MVN(0, Q) \quad (11)$$

where τ_t is the trend, which is driven by long-run factors: the number of households (P_t), the natural interest rate (R_t) and housing stock (S_t). α_t^p , α_t^r and α_t^s are corresponding coefficients that can vary over time, following random walk processes. $X_t = [c_t \ u_t \ l_t]'$ is a vector of cyclical variables, including the unobserved cyclical component of HPI (c_t), the unemployment rate (u_t) and the rate of mortgage growth (l_t). The cyclical variables are assumed to follow a VAR(1) process, with the variance-covariance matrix, Σ , being a diagonal matrix. The VAR model of the cyclical variables is unconstrained, allowing all cyclical variables to interact with each other.

In the benchmark model, I choose two cyclical indicators and three trend indicators, as shown in Figure (7). The indicator variables for the trend component are selected to capture the long-run demand and supply factors in housing markets, based on the housing literature (Geng, 2018; see e.g. Muellbauer and Murphy, 1997). I use the number of households to proxy demand for housing. The second trend indicator summarizes changes in financing conditions for the Irish housing market. Last, I use housing stock to account for housing supply in Ireland. For informing cyclical movements in the HPI ratio, I select the unemployment rate as the business cycle indicator and mortgage lending growth as the financial cycle indicator. Both cyclical indicators are stationary, as suggested by the augmented Dickey-Fuller test. In a robustness test, I also consider a different set of cyclical indicators, such as the inflation rate and new residential property completions.⁹ All variables are in log terms, except for the natural interest rate and growth rates. Data are demeaned and normalized to have a standard deviation of one before being used in the empirical analysis.

3 Empirical results

This section reports the empirical results of the multivariate model with fixed coefficients. To do that, I set the variances of ζ_{1t} , ζ_{2t} and ζ_{3t} in Equation (8) - (10) to zero, which changes time-varying coefficient model into the standard fixed coefficient UC model.

Empirical results are summarised in Table (2) and Figure (3). First, as shown in the left panel of Figure (3), the estimated trend of the HPI ratio is rising smoothly, relative to cycles, over time. Based on statistical significance, the trend is primarily driven by two structural

⁸ The state-space form of the multivariate UC model can be found in the appendix.

⁹ In Table (4), I summarize the definition of all auxiliary variables and data sources.

drivers in the model. Housing demand driven by population growth, on the one hand, has a significant positive impact on the HPI trend; the natural interest rate, on the other hand, exerts a negative effect. The magnitudes of those elasticities are also economically non-trivial. A one percentage change in housing demand leads to 0.17% increase in the long-run HPI ratio, while a one-percent fall in the natural interest rate could result in a 3% increase in the HPI trend. Given that the natural interest rate in Ireland falls about 10 percentage points in our sample - from 8% in 1992 to -2% in 2021, the majority of increases in the HPI ratio observed in the data can be explained by the falling natural interest rate. Variance decomposition based on the estimated coefficients confirms that 84% of the variance of the HPI trend can be explained by the natural interest rate, while 13% of the variance can be attributed to housing demand factor, and only 3% is explained by housing stock. Miles and Monro (2019), using a different methodology, also find that the rise in house prices relative to incomes between 1985 and 2018 in the UK can be accounted for by an unexpected decline in the real risk-free interest rate observed over the period. Despite these interesting findings, it is worth noting that the trend estimate is subject to substantial uncertainty, as the confidence bands are very wide.¹⁰

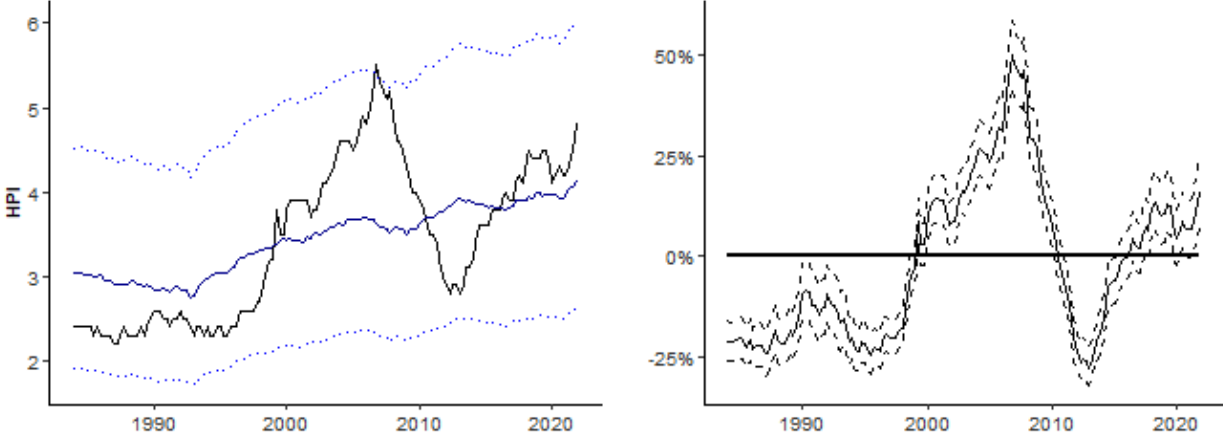


Figure 3. Decomposition of HPI using Kalman filter

Second, the estimated VAR model of the cyclical system shows highly persistent dynamics. Diagonal coefficients (a_{11} , a_{22} , a_{33}) of all cyclical variables are highly significant and close to one. In terms of the off-diagonal coefficients, I find that the unemployment rate has a significant negative correlation with the cyclical component of the HPI ratio (a_{12}), while mortgage growth shows a significant positive correlation with the cyclical component of the HPI ratio (a_{12}). The diagnostic plots shown in Figure (8) also confirm that residuals of HPI are white noises, but residuals of unemployment and mortgage growth still have significant coefficients on the lags. This result indicates that a VAR(1) model for the cyclical dynamics might be too restrictive to capture the persistence in the cyclical indicators.¹¹

In the right panel of Figure (3), I plot the estimated HPI cycles. As argued in the introduction, HPI is a better indicator of housing boom-bust cycles than the house price itself. The estimated HPI cycles are consistent with the historical experiences of the Irish

¹⁰ In Figure (3), we plot the confidence interval, based on bootstrapping 1000 random draws on estimated trend coefficients, and the percentiles of the confidence band are 40% and 60%.

¹¹ In a robustness test, I relax this assumption to allow a VAR(2) specification for the cyclical model.

Table 2. Estimation Results of Multivariate UC Model

	estimate	std.error	conf.low	conf.up
α_P	0.1702	0.0903	-0.0068	0.3472
α_R	-0.0316	0.0059	-0.0431	-0.0200
α_S	-0.0778	0.0895	-0.2531	0.0976
a_{11}	0.9359	0.0241	0.8887	0.9831
a_{21}	0.1579	0.0668	0.0269	0.2889
a_{31}	-0.9652	0.4159	-1.7803	-0.1501
a_{12}	-0.0134	0.0047	-0.0227	-0.0042
a_{22}	1.0041	0.0133	0.9781	1.0300
a_{32}	-0.1269	0.0780	-0.2797	0.0259
a_{13}	0.0105	0.0028	0.0050	0.0161
a_{23}	-0.0630	0.0092	-0.0810	-0.0450
a_{33}	0.8161	0.0476	0.7228	0.9093
σ_ϵ	0.0001	0.0001	0.0000	0.0003
q_{33}	0.0008	0.0002	0.0005	0.0012
q_{44}	0.0099	0.0011	0.0076	0.0121
q_{55}	0.2325	0.0270	0.1795	0.2855

Note:

Log-likelihood: 327.1, AIC: -627.3. AIC is calculated by using the small sample size corrected AIC (Cavanaugh and Shumway, 1997). Confidence intervals are calculated at 0.05 and 0.95 percentiles via Hessian methods.

housing market. I identify two large housing cycles in the sample period. The first boom-bust cycle started from the economic turmoil in the 1980s, recovered during “Celtic Tiger” years and ended in 2007 just before the GFC. The second cycle started from the sharp decline in 2008 and turned to recover around 2013. The most recent house price recovery was briefly disrupted by the COVID-19 outbreak, but continues to rise until the end of the sample period. Due to the limits of the sample period, we don’t observe the full cycles, but each housing boom-bust cycle lasts more than ten years. In addition, HPI cycles in Ireland also exhibit large deviations from its trend. In both housing downturns, I observe 25% deviations below the trend, while in the housing boom of the 2000s deviations from the trend were as large as 50%.

4 Robustness

In this section, I will discuss the robustness of the empirical result from the last section. The robustness tests will be conducted in four broad areas. First, I introduce additional drivers of the HPI trend. Second, alternative cyclical settings of HPI are considered. Third, I test over-identification assumptions. Last but not least, potential structural breaks are checked either explicitly or using time-varying coefficient estimation.

4.1 Credit driver of the HPI trend

In the benchmark model, the trend equation includes housing supply/demand and the natural rate of interest which captures the cost of credit. There is broad historical evidence showing that shifts in credit availability have fueled asset price movements (Schularick and Taylor, 2012), including house prices. As a consequence, it is justified to include a measure

of credit availability into the trend model. However, tracking mortgage credit standards is difficult as most nations lack data on time variation in non-price terms of mortgage credit. For the US, Duca et al. (2011) proxy mortgage standards using American Housing Survey data on the median LTV for first time home-buyers. To track UK mortgage conditions, Fernandez-Corugedo and Muellbauer (2006) use micro data on LTVs and DTI ratios for first-time buyers. Kelly et al. (2015) find that LTV and DSTI ratios for first-time borrowers moved together in the recent Irish housing boom and bust.

In this paper, I follow McQuinn and O'Reilly (2006), which document evidence of the existence of a long-run relationship between actual house prices and the amount individuals can borrow in Irish property market. The credit condition measure used in this paper is based on the idea that the amount lent by a mortgage institution to an individual is dependent on current disposable income and interest rates. Based on this observation, they back out how much a financial institution would lend an individual given plausible assumptions regarding the fraction of income that goes to mortgage repayments and the duration of the mortgage using a standard annuity formula. I add this series into my trend model and simplify the housing demand/supply factors by using the ratio between the stock of housing and the number of the population of household formation age (25 - 44 years old). This ratio captures the supply/demand imbalance in the Irish housing market.

As a result, the HPI trend model is as follows,

$$\tau_t = \alpha_t^i Imbalance_t + \alpha_t^r R_t + \alpha_t^c Credit_t + \mu_t, \quad \mu_t \sim N(0, \sigma_\mu^2) \tag{12}$$

where τ_t is the trend, which is driven by long-run factors: supply/demand imbalance ($Imbalance_t$), the natural interest rate (R_t) and credit availability ($Credit_t$).

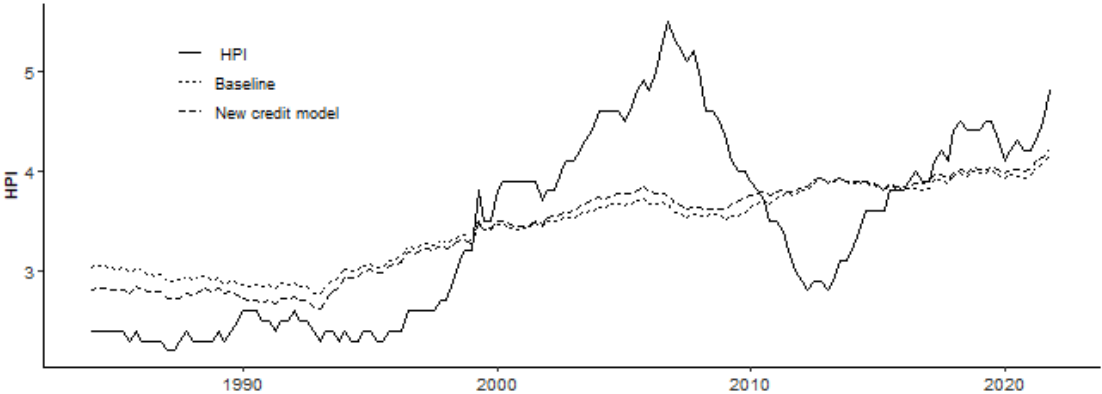


Figure 4. Alternative trend model

Empirical results from this new trend model also broadly similar to the benchmark model. As shown in Figure (4), both estimated trends are very similar, except for periods at the beginning of the sample and during the property boom before the GFC. It is also worthy noting that including credit measure into the trend model does not fundamentally change the variance decomposition results. Based on the estimated coefficients form the new model, 79% of the variance of the HPI trend can be explained by the natural interest rate, while 19% of the variance can be attributed to the credit condition. As a result, only 1.4% of the variance of the HPI trend is explained by housing market imbalances.

4.2 Alternative cyclical model settings

In the baseline model, I use the unemployment rate and the new mortgage lending growth as cyclical indicators in VAR to identify cyclical component of HPI. To show the result is not specific to these drivers, I replace them with inflation as the business cycle driver and residential property completions as the cyclical indicator of the housing market. In addition, I also consider a more general VAR setting with two lags for the cyclical dynamics. VAR(2) model would help to resolve the serial correlation in residuals identified in Figure (8).

Robustness results regarding to cyclical settings are shown in the left panel of Figure (5). New set of cyclical indicators has not a significant impact on the estimated trend. The estimated trend follows the benchmark closely. VAR(2) model, on the other hand, seems to change the trend in the early period of the sample, while the estimated trend is very close to the benchmark at the second half of the sample. Figure (9) shows that longer lag structure in the VAR setting does help to reduce the serial correlation in the residual of unemployment, but the residuals of new lending series still show some significant serial autocorrelation.

4.3 Over-identification assumptions

In the baseline model, there are 6 unobserved state variables, while 27 restrictions on the coefficients are imposed. This leads the baseline model to be over-identified. In this section, I test three alternative models with exact-identified restrictions against the baseline. In particular, I relax 6 restrictions at a time, and use the likelihood ratio test as well as the Akaike Information Criterion (AIC) (Cavanaugh and Shumway, 1997) to evaluate the over-identification assumptions.

As shown in Matrix equation (14) in Appendix, the transition matrix in the state equation has 27 restrictions. I could relax 6 restrictions to make the system exactly identified. In the following over-identification tests, I retain the assumption that trend coefficients are not influenced by the cyclical variables, but I test three alternative models where 6 out of 9 zero parameters in the lower-left corner of the transition matrix are relaxed. Those non-zero coefficients would allow for changes in the HPI trend to affect housing cycles. This renders more general settings compared to the baseline model. In the first alternative, I allow the HPI trend to affect the cyclical component of HPI and the unemployment rate. In the second model, HPI trend can affect the cyclical component of HPI and the new lending growth. In the last setting, HPI trend is allowed to influence the unemployment rate and the new lending growth.

Table 3. Over-identification tests

Model	Log Likelihood	AIC	Likelihood Ratio	P-value
1	314.8	-589.83	2.28	0.89
2	314.7	-589.70	2.14	0.91
3	314.8	-589.83	2.28	0.89

Note: The benchmark model has a Log-likelihood of 313.6 and AIC of -600.5. AIC is calculated by using the small sample size corrected AIC (Cavanaugh and Shumway, 1997)

Over-identification test results are summarized in Table (3). Although alternative models achieve slightly higher log likelihood, but, based on the likelihood ratio tests, all three

models are not significantly different from the baseline model. When assessing the overall performance of the richer models against the benchmark, results show that AICs of richer models are even slightly larger than the benchmark model, indicating a overall worst forecasting performance.

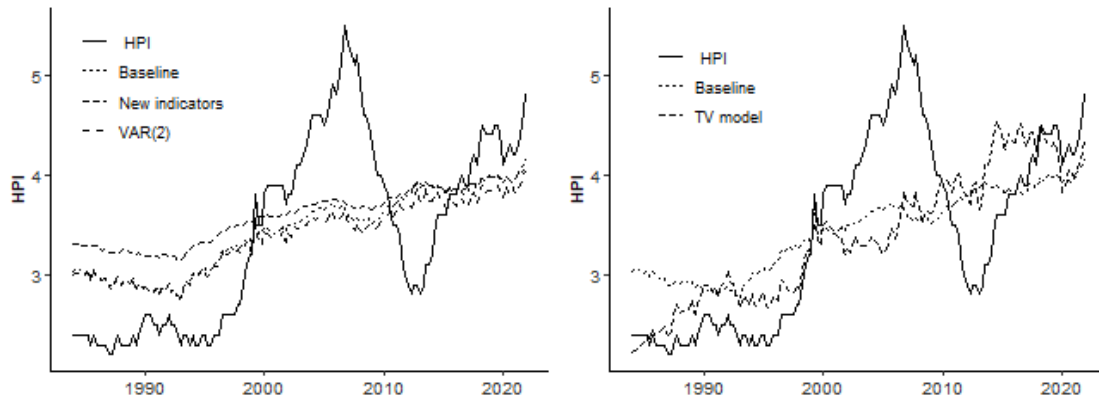


Figure 5. Robustness tests

4.4 Structural breaks

In this section, I explore the UC model with time-varying coefficients. As discussed in introduction, one of main criticisms of existing econometric models is that they assume that the estimated relationship is stable over time. This assumption is particularly problematic when applying to housing markets. The price elasticity of housing supply and demand, for example, vary over time due to a wide range of reasons, such as changes in the tax and housing policies. To address potential structural changes in the housing market, state-space modeling provides a framework allowing for estimating time-varying coefficients.

In Figure (6), three time-varying coefficients of the drivers to the trend are shown and some interesting patterns emerge.

Regarding to elasticity to housing market fundamentals, housing demand driven by the household formation is significantly positive, but neither HPI trend elasticities to demand and supply show significant variations over time. On the other hand, the time-varying interest rate coefficient is only significantly negative in the beginning of the sample period. It becomes insignificantly from zero around 2000, indicating a major structural break in the transmission mechanism of the interest rate shock to the long-run house price. The likely economic interpretation of this finding is that, during this period, the economy finds itself in a secular stagnation (Eggertsson et al., 2016), where real demand for housing become less responsive to the real interest rates.

Despite the interesting patterns found in the time-varying coefficient model, the trend estimate of TV model, shown in the right panel of Figure (5), exhibits a more volatile dynamics. This feature makes the trend estimate less attractive from the policy point of view.

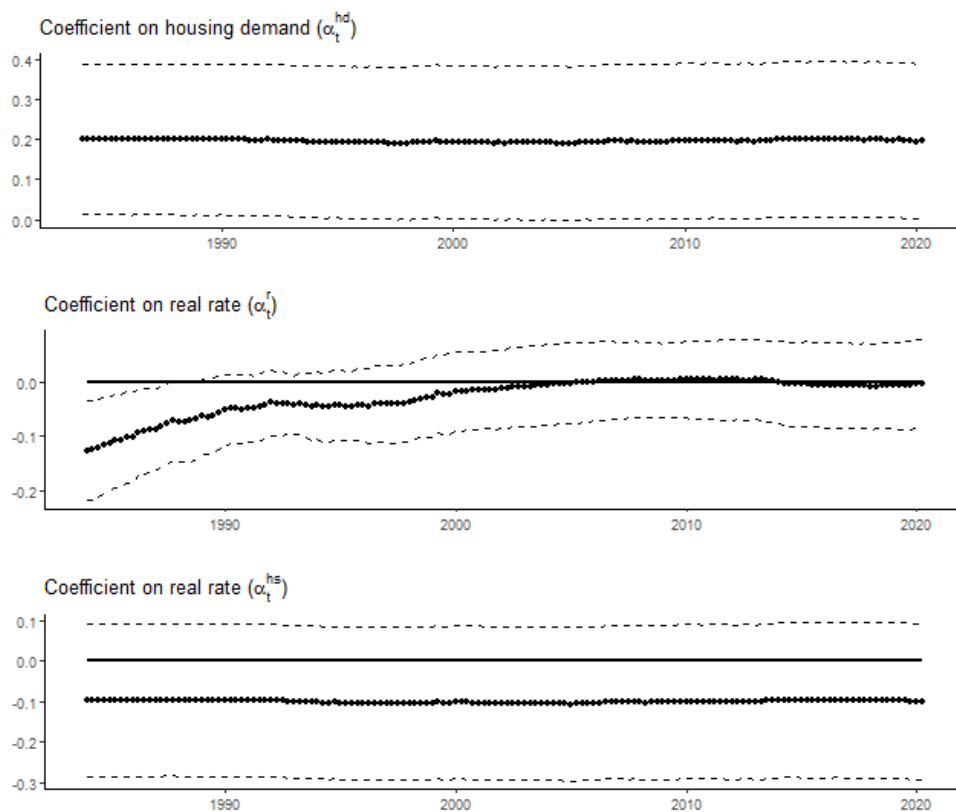


Figure 6. Robustness tests

5 Policy discussion

In this section, I discuss potential policy uses of the estimated trend and cycles of HPI based on the baseline model.

First, the estimated trend of HPI is useful for guiding structural macroprudential instruments, such as the loan-to-income ratio (LTI). When implemented in a structural fashion, an LTI limit is not expected to respond regularly to movements in the financial cycle, unlike other macroprudential instruments such as the Counter-cyclical Capital Buffer. As long as its calibration is consistent with the long-run fundamental level, LTI will work automatically to mitigate cyclical fluctuations in credit. Where long-run, structural changes in the equilibrium HPI are identified in a methodology such as this, they can be used as one motivating factor in analysing the merits of recalibration of borrower-based measures. As a result, estimated changes in the HPI trend is informative for calibrating the change in LTI. Since the implementation of mortgage measures in 2015, LTI restriction has been set at 3.5 in Ireland. During the same period time, the HPI trend has risen by 8% from 3.8 in 2015 to 4.1 by the end of 2021. This increase in trend HPI suggests that a flat LTI calibration has been slowly become more binding in Ireland since 2015, even before accounting for the impact of cyclical forces on access to the mortgage market. Based on the estimates in this paper, the Central Bank's recalibration from 3.5 to 4 for First Time Buyer LTI ratios in October 2022 is consistent with the costs of the previous calibration becoming greater over time, as a higher underlying HPI implied rising difficulties in accessing the housing market at a fixed 3.5 LTI limit"

Second, deviations of HPI from the long-run trend can be used to gauge the build-up of housing vulnerability. The Estimated cycles in Figure (3) show that since 2014 cyclical house price has been rising, although it was interrupted by the outbreak of the COVID-19 pandemic. The latest observations suggest that the housing vulnerability starts building up. This is, however, still far below the most acute phase of housing market overvaluation experienced before the GFC in 2008.¹²

6 Conculsion

This paper proposes a novel method to estimate the trend and cycle component of the house price to income ratio with multivariate state-space models. The main contribution of the paper is to enrich the multivariate setting by introducing auxiliary variables for both unobserved trend and cyclical components. As a result, the estimated trend and cycles are identified more from data, less from the parametric assumptions. This also makes the empirical results more economically interpretable.

Furthermore, empirical findings support the proposed decomposition model to be a useful tool for macroprudential policy. On the one hand, the estimated cycles of HPI provides a credible measure of the built-up of vulnerability in the housing market. Historically speaking, when the estimated cycle is significantly deviated from the trend, it likely signals a upcoming downturn. On the other hand, the estimated trend of the HPI ratio can be used to guide the calibration of mortgage measures such as LTI. Where long-running, slow-moving forces are estimated to be increasing the underlying trend in HPI, this can act as a motivating factor for recalibration of instruments such as LTI limits, given that constant LTI limits become more binding over time when trend HPI is growing.

¹² Central Bank of Ireland currently uses a suite of models to assess the cyclical development in the housing market. My estimate is at the upper range suggested by the Central Bank modelling.

References

- Abraham, J.M., Hendershott, P.H., 1994. Bubbles in Metropolitan Housing Markets (NBER Working Papers No. 4774). National Bureau of Economic Research, Inc.
- Aikman, D., Kiley, M., Lee, S.J., Palumbo, M.G., Warusawitharana, M., 2017. Mapping heat in the U.S. financial system. *Journal of Banking & Finance* 81, 36–64.
- Baxter, M., King, R.G., 1999. Measuring Business Cycles: Approximate Band-Pass Filters For Economic Time Series. *The Review of Economics and Statistics* 81, 575–593.
- Beveridge, S., Nelson, C.R., 1981. A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the “business cycle”. *Journal of Monetary Economics* 7, 151–174.
- Cavanaugh, J.E., Shumway, R.H., 1997. A bootstrap variant of AIC for state-space model election. *Statistica Sinica* 7, 473–496.
- Christiano, L.J., Fitzgerald, T.J., 1999. The Band pass filter (Working Papers (Old Series) No. 9906). Federal Reserve Bank of Cleveland.
- Clark, P., 1987. The cyclical component of u. S. Economic activity. *The Quarterly Journal of Economics* 102, 797–814.
- Duca, J.V., Muellbauer, J.N., Murphy, A., 2011. House prices and credit constraints: making sense of the U.S. experience (Working Papers No. 1103). Federal Reserve Bank of Dallas.
- Eggertsson, G.B., Mehrotra, N.R., Summers, L.H., 2016. Secular Stagnation in the Open Economy (NBER Working Papers No. 22172). National Bureau of Economic Research, Inc.
- Fernandez-Corugedo, E., Muellbauer, J., 2006. Consumer credit conditions in the United Kingdom (Bank of England working papers No. 314). Bank of England.
- Galán, J.E., Mencía, J., 2018. Empirical assessment of alternative structural methods for identifying cyclical systemic risk in Europe (Working Papers No. 1825). Banco de España.
- Galati, G., Hindrayanto, I., Koopman, S.J., Vlekke, M., 2016. Measuring financial cycles with a model-based filter: Empirical evidence for the United States and the euro area (DNB Working Papers No. 495). Netherlands Central Bank, Research Department.
- Geng, Ms.N., 2018. Fundamental Drivers of House Prices in Advanced Economies (IMF Working Papers No. 2018/164). International Monetary Fund.
- Girouard, N., Kennedy, M., Noord, P. van den, André, C., 2006. Recent House Price Developments: The Role of Fundamentals (OECD Economics Department Working Papers No. 475). OECD Publishing.
- Hamilton, J.D., 2018. Why You Should Never Use the Hodrick-Prescott Filter. *The Review of Economics and Statistics* 100, 831–843.
- Harvey, A., 1991. *Forecasting, structural time series models and the kalman filter*. Cambridge University Press.
- Harvey, A.C., Jaeger, A., 1993. Detrending, Stylized Facts and the Business Cycle. *Journal of Applied Econometrics* 8, 231–247.
- Hodrick, R.J., 2020. An Exploration of Trend-Cycle Decomposition Methodologies in Simulated Data (NBER Working Papers No. 26750). National Bureau of Economic Research, Inc.
- Hodrick, R.J., Prescott, E.C., 1997. Postwar U.S. Business Cycles: An Empirical Investigation. *Journal of Money, Credit and Banking* 29, 1–16.
- Kelly, R., McCann, F., O’Toole, C., 2015. Credit conditions, macroprudential policy and house prices (Research Technical Papers No. 06/RT/15). Central Bank of Ireland.

- Kennedy, G., O'Brien, E., Woods, M., 2016. Assessing the sustainability of Irish residential property prices: 1980Q1-2016Q2 (Economic Letters No. 11/EL/16). Central Bank of Ireland.
- Lang, J.H., Welz, P., 2018. Semi-structural credit gap estimation (Working Paper Series No. 2194). European Central Bank.
- Laubach, T., Williams, J.C., 2003. Measuring the Natural Rate of Interest. *The Review of Economics and Statistics* 85, 1063–1070.
- Lowe, P., Borio, C., 2002. Asset prices, financial and monetary stability: exploring the nexus (BIS Working Papers No. 114). Bank for International Settlements.
- Lucas, A., Koopman, S.J., 2005. Business and default cycles for credit risk. *Journal of Applied Econometrics* 20, 311–323.
- McQuinn, K., O'Reilly, G., 2006. Assessing the Role of Income and Interest Rates in Determining House Prices (Research Technical Papers No. 15/RT/06). Central Bank of Ireland.
- Miles, D., Monro, V., 2019. UK house prices and three decades of decline in the risk-free real interest rate (Bank of England working papers No. 837). Bank of England.
- Morley, J.C., Nelson, C.R., Zivot, E., 2003. Why Are the Beveridge-Nelson and Unobserved-Components Decompositions of GDP So Different? *The Review of Economics and Statistics* 85, 235–243.
- Muellbauer, J., Murphy, A., 1997. Booms and busts in the UK housing market. *Economic Journal* 107, 1701–27.
- Nelson, C.R., Plosser, C.I., 1982. Trends and random walks in macroeconomic time series : Some evidence and implications. *Journal of Monetary Economics* 10, 139–162.
- O'Brien, M., Velasco, S., 2020. Unobserved components models with stochastic volatility for extracting trends and cycles in credit (Research Technical Papers No. 09/RT/20). Central Bank of Ireland.
- Philipponnet, N., Turrini, A., 2017. Assessing House Price Developments in the EU (European Economy - Discussion Papers 2015 - No. 048). Directorate General Economic; Financial Affairs (DG ECFIN), European Commission.
- Schularick, M., Taylor, A.M., 2012. Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870-2008. *American Economic Review* 102, 1029–1061.
- Schweppe, F., 1965. Evaluation of likelihood functions for gaussian signals. *IEEE transactions on Information Theory* 11, 61–70.

Appendix

A.1 Estimating the natural rate of interest

To estimate the natural rate of interest used in this paper, we use a methodology similar to the approach of Laubach and Williams (2003).¹³ We treat the equilibrium interest rate as an unobservable state variable, and derives estimates of the series using a filtering approach.

Formally, we can represent the approach in state space form,

$$r_t = \mu_t + \phi_t + \sum_{i=1}^4 \pi_{t-i} + \sum_{i=1}^4 y_{t-i} + e_t \quad (13)$$

$$\mu_t = \mu_{t-1} + \gamma_{t-1} + v_t \quad (14)$$

$$\gamma_t = \gamma_{t-1} + \xi_t \quad (15)$$

where r_t is the real rate of interest, μ_t is a trend component that follows a smooth deterministic process, π_t is a measure of inflation and y_t is the real growth rate of output. The error components are all considered to be distributed as $x \sim i.i.d.N(0, \sigma_x^2)$, where $x \in \{e, v, \xi\}$. The cyclical component, ϕ_t , follows a first-order stochastic process and can be represented as

$$\phi_t = \rho \cos(\lambda_c) \phi_t + \rho \sin(\lambda_c) \phi_{t-1}^* + \eta_t \quad (16)$$

$$\phi_t^* = -\rho \sin(\lambda_c) \phi_{t-1} + \rho \cos(\lambda_c) \phi_{t-1}^* + \eta_t^* \quad (17)$$

where ρ is a dampening factor such that $0 \leq \rho \leq 1$, λ_c is the frequency of the cycles, and $z \sim i.i.d.N(0, \sigma_z^2)$, where $z \in \{\eta, \eta^*\}$.

To calculate the model, we take data from the ECB Area Wide Model (AWM) database, updated to 2021. Nominal interest rates are represented by the Euribor 3-month rate, in percent per annum terms. Output is the chain-linked volume series for euro area GDP, which is calendar and seasonally adjusted. Inflation is measured as the annual growth rate of the euro area HICP.

The parameters of the model are estimated via maximum likelihood, while a diffuse Kalman filter is used to write the likelihood function in prediction-error form.

¹³ As the natural rate of interest used in our state-space model is itself an estimate. I perform a bootstrapping exercises with 500 simulated natural rate series. The result of the bootstrapping shows that uncertainty around the estimated natural rate does not change the point estimate of the HPI trend but increase the confident bands.

A.2 Multivariate State-space Model

$$\begin{bmatrix} y_t \\ u_t \\ l_t \end{bmatrix} = \begin{bmatrix} P_t & R_t & S_t & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_t^P \\ \alpha_t^R \\ \alpha_t^S \\ c_t \\ u_t \\ l_t \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ 0 \\ 0 \end{bmatrix} \quad (18)$$

$$\epsilon_t \sim N(0, \sigma_\epsilon^2) \quad (19)$$

$$\begin{bmatrix} \alpha_t^P \\ \alpha_t^R \\ \alpha_t^S \\ c_t \\ u_t \\ l_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{11} & a_{12} & a_{13} \\ 0 & 0 & 0 & a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 & a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \alpha_{t-1}^P \\ \alpha_{t-1}^R \\ \alpha_{t-1}^S \\ c_{t-1} \\ u_{t-1} \\ l_{t-1} \end{bmatrix} + \begin{bmatrix} \tilde{\zeta}_{1t} \\ \tilde{\zeta}_{2t} \\ \tilde{\zeta}_{3t} \\ \tilde{\zeta}_{4t} \\ \tilde{\zeta}_{5t} \\ \tilde{\zeta}_{6t} \end{bmatrix} \quad (20)$$

$$\Xi_t \sim MVN(0, Q) \quad (21)$$

In the observation equation (13), y_t is observed HPI, which is driven by long-run factors and the cyclical component (c_t). The unemployment rate (u_t) and the rate of mortgage growth (l_t) are two observable cyclical indicators used in the model. For the model to support time-varying coefficients in HPI trend, I put data on trend driving factors (P_t, R_t, S_t) into the coefficient matrix, while making the trend coefficients ($\alpha_t^P, \alpha_t^R, \alpha_t^S$) to be unobserved states.

In state equation (15), besides the long-run trend coefficients, unobserved state-space also includes unobserved cyclical component (c_t). To allow cyclical component to be affected by other cyclical indicators, I include them into the state space.

Tables and Figures

Table 4. Data sources

	Source	Details
House price-to-income ratio	CSO	Nominal house price to nominal disposable income ratio
Trend indicators		
Household formation cohort	CSO (Census, population projection)	Series based on CSO population projections (Persons in April) (Thousand) by Sex, Age Group (25-44)annual. Quarterly data interpolated by spreading the annual change evenly between the quarters.
Natural rate	Central Bank of Ireland	Central bank estimation based on state-space modelling
Housing Stock	CSO (Census)	Number of dwellings in Census, plus quarterly completions (depreciated)
Cyclical indicators		
Credit availability	@McquinnOReilly	Data is extended by following the parameters in McQuinn and O'Reilly's Affordability Model.
Unemployment rate	OECD	Data is interpolated from annual figures pre-1998
New lending growth	CBI	Quarterly Financial Accounts. Household balance sheet sum of loans and securities other than shares.
inflation rate	CSO	Harmonised Index of Consumer Prices
New residential completions	Department of housing, CSO	Total ESB connections 1980-2011Q3, CSO completions thereafter

Note: CSO: Central Statistic Office of Ireland. CBI: Central Bank of Ireland

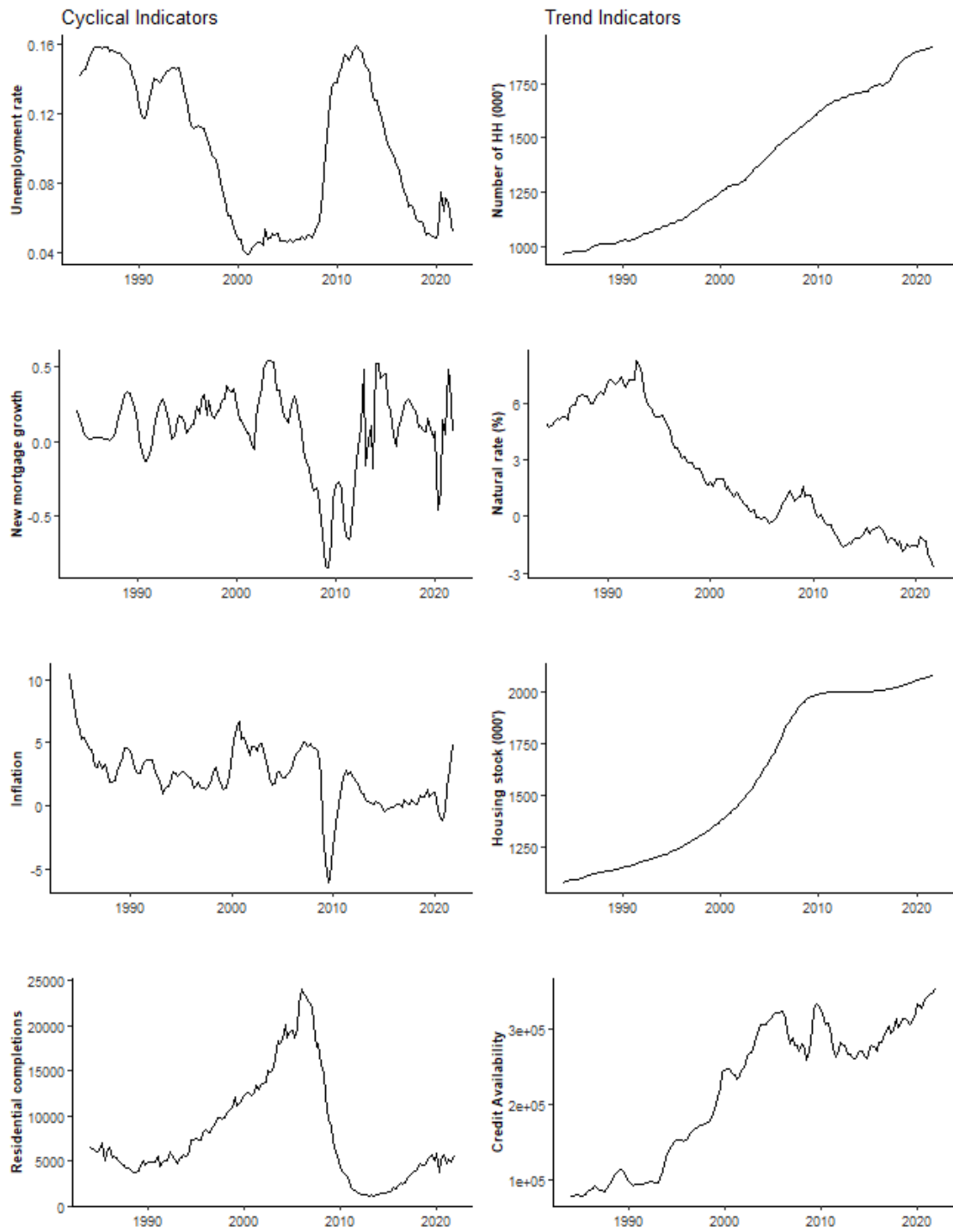


Figure 7. Cyclical and trend indicators

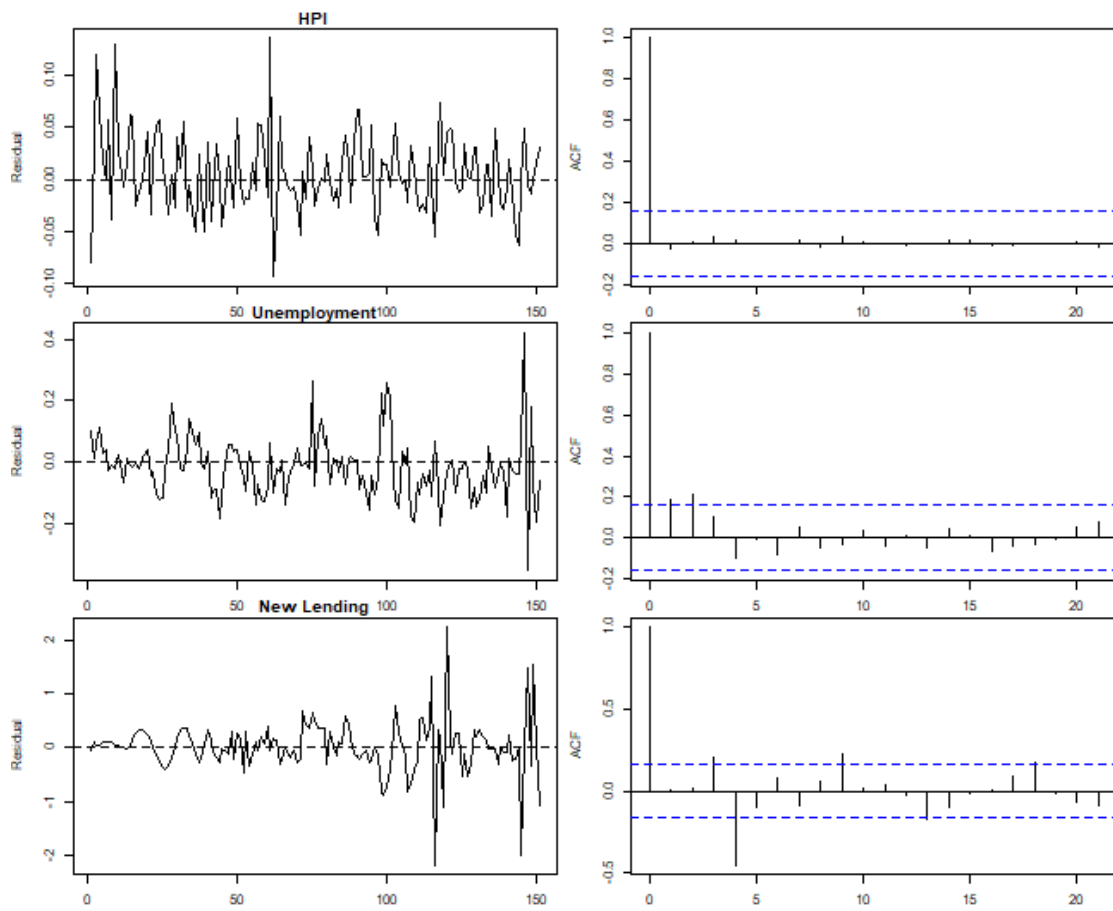


Figure 8. Diagonal plot for the baseline model

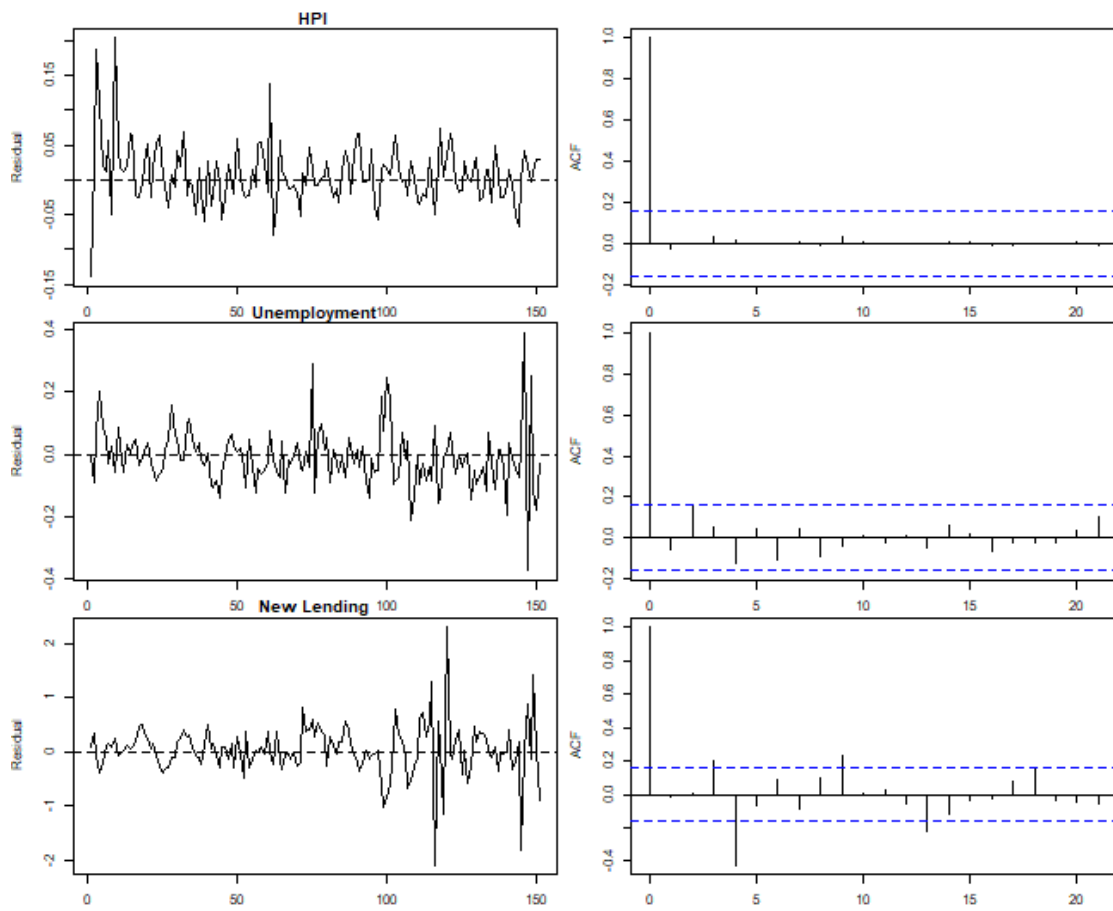


Figure 9. Diagonastic plot for VAR(2) model