Credit Risk to NBFIs, Liquidity Risk to Banks: A Conceptual Framework*

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Abstract

Banks and non-bank financial intermediaries (NBFIs) coexist, in a symbiotic relationship where credit risk migrates to NBFIs but liquidity risk stays with banks. We provide a theoretical rationale for this "transformed" intermediation, that relies on three key features: (i) a funding advantage to banks from deposits relative to equity financing, (ii) a bank advantage in providing contingent liquidity relative to NBFIs, and (iii) "coarse" regulatory capital requirements, optimally set to trade off the cost of deposit bail outs with the benefits of economic activity, but lacking in adequate variability with respect to subordination and credit lines. We show that these frictions lead to "regulatory arbitrage" in the financial sector that seeks to warehouse credit risks away from banks while still harnessing bank funding advantages. We discuss how the level of capital requirements, even if coarse, can optimally factor in the tradeoff between efficiency and financial stability, especially from credit-line provision by banks to NBFIs.

Keywords: nonbank financial intermediation, nonbanks, shadow banking, bank regulation, regulatory arbitrage, systemic risk, subordination, credit lines, derivatives margin.

JEL Codes: G01, G21, G23, G28

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1 Introduction

The growth and size of nonbank financial intermediaries (NBFIs) relative to the banking sector is of great current interest to market participants, academics, and regulators. In particular, NBFIs have become the dominant segment of the intermediation industry, with their global financial assets representing a larger (and growing) share with respect to banks (Figure 1). The conventional narrative explaining this trend suggests that NBFIs are substituting for banks in the provision of intermedation services. A closer examination, however, shows that the growth of NBFIs does not occur independently of banks, but rather it is *supported* by banks, via the provision of loans and credit lines, necessary inputs to their intermediation activities. This is clearly evident in the data for U.S. banks, portrayed in Figure 2 and Figure 3, showing not only a growing trend in loan and credit lines to NBFI obligors, but in fact a shift in business focus by banks away from its traditional core, nonfinancial corporations, towards NBFIs.

In a series of recent papers and short articles, including Acharya, Cetorelli, and Tuckman (2026a, b), we used financial accounting data, case studies, and empirical work to document the existence of important complementarities between banks and NBFIs, and to put forward a *transformation* view of the relationship between the bank and NBFI sectors. More specifically, financing businesses have transformed in response to a combination of post-GFC regulations, which raised the costs to banks of holding junior credit risks on balance sheet, and the funding and liquidity advantages of banks, from their deposit franchises and special access to central bank support. As a result, junior credit risks have migrated to NBFIs, senior credit risks to banks, and contingent liquidity risks, like credit lines, are offered by banks to NBFIs.

The purpose of this paper is to provide a theoretical foundation for this transformation of intermediation risks and activities between banks and NBFIs. The foundation we provide is based on three key pillars: First, banks have an intrinsic advantage in sourcing stable

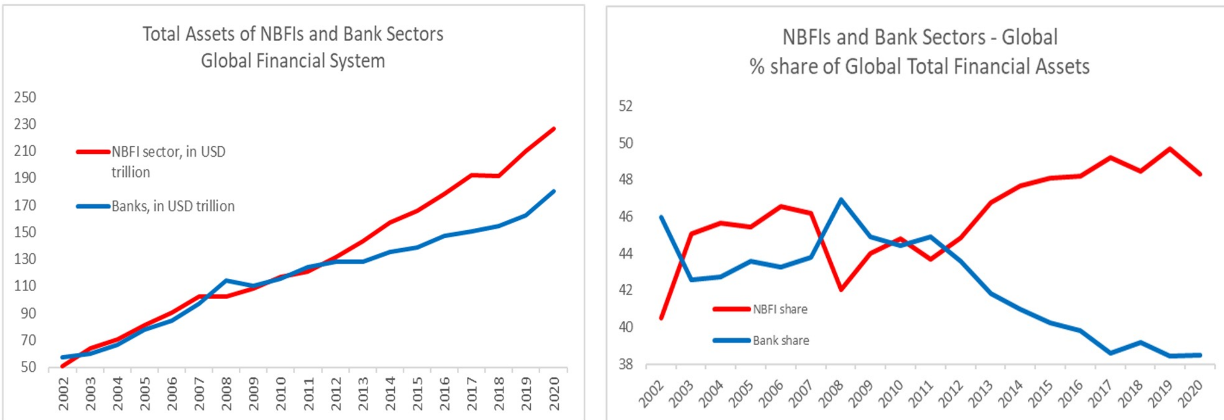


Figure 1: Growth of NBFIs and Domination over banks post GFC

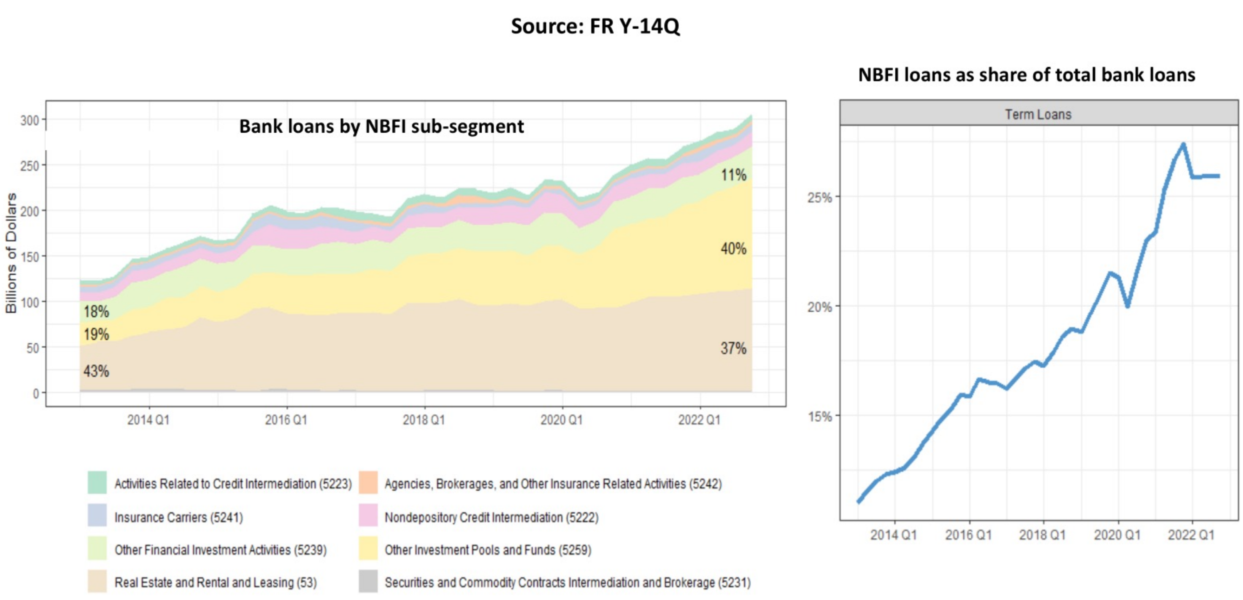


Figure 2: Large Increase in Bank Loans to NBFIs Post GFC

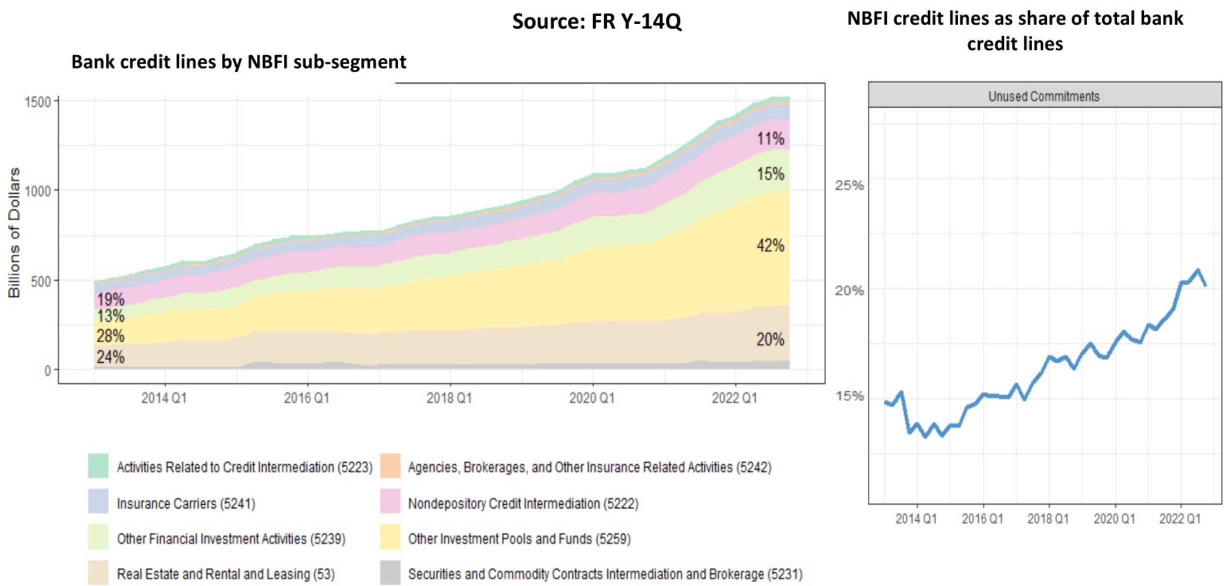


Figure 3: Large Increase in Credit Lines to NBFIs Post GFC

deposit funding relative to raising equity, the result of exclusive access to deposits that provide transactional services to savers and due to official backstops (deposit guarantees and access to central bank liquidity facilities). Second, NBFIs can provide intermediation services, but lack the intrinsic funding advantages that banks have in providing contingent liquidity (for example, credit lines). Third, intermediation activity of banks is regulated, notably through capital requirements (that are not applied to NBFIs), to trade off the costs associated with providing official backstops against the benefits of greater economic activity.

Under these plausible assumptions, the interplay of banks' and regulator's optimization leads to the conditions for the natural emergence of a set of (unregulated) non-bank intermediaries, and to the necessary existence of a symbiotic relationship between these NBFIs and the banks. In particular, subordination of credit risk by NBFIs helps relax bank capital requirements, and banks provide senior and contingent funding to manage the liquidity risk of the NBFIs. In this paper, we illustrate these mechanisms theoretically.

The basic setup of our conceptual framework is that a unit loan, which can pay its promised return or default on the full repayment of principal, is funded by a combination of deposits, bank equity, and a junior tranche held by an NBFI, which is contractually subordinated to bank equity. The key starting assumption of the model is that banks, optimizing the return to bank equity, can raise funds relatively cheaply through deposits, but regulatory capital requirements limit the extent of deposit funding. The next assumption is that capital requirements are relatively "coarse": they are optimally set to trade off the cost of deposit bail outs with the benefits of economic activity, but lack adequate variations with respect to subordination and credit lines. In other words, a "neutral" regulatory design – one where banks have no incentives to subordinate – is too complex and infeasible to implement. As a result, there are conditions where the limit to cheap deposit funding can be increased by subordination, that is, by having an NBFI partially fund the loan through a junior claim. In other words, the first-loss protection of a junior claim of a particular size reduces the required amount of bank capital sufficiently so as to allow for an increase in the quantity of deposit funding.

The government or regulatory sector in the model not only sets capital requirements, but also sets a policy with respect to bailing out depositors. It is assumed that the bailout policy extends only to bank depositors because of the possibility of bank runs and the disruption of payments and settlements in the economy, and does not extend to NBFIs. These bailouts are priced by all market participants, which means, for example, that more generous bailout policies lower the rate that bank equity needs to pay to attract deposit funding. In the first stage of the development of the model, capital requirements are taken as exogenous. In the second stage, discussed below, the government can to some extent optimize over capital requirements. More precisely, the government can change the level of the capital requirements, but not their coarseness, i.e., the government cannot increase their sensitivity to subordination and credit lines so as to remove the incentive for subordination.

The model assumes that banks can always raise liquidity through stress periods, because of their special role and place in the financial system, granted by exclusive access to raising deposits and to official backstops, while NBFIs cannot. The model also assumes that the NBFI's holding of the junior claim is accompanied by some ongoing responsibilities of the

NBFI to the ultimate borrower regarding the borrower's liquidity risk (as detailed below). Together, these two assumptions imply a downside to subordination: in a liquidity crisis, if an NBFI has no access to liquidity, in particular, a bank credit line, then the NBFI cannot support the ultimate borrower, and the loan defaults as a casualty of that liquidity crisis.

This crucial feature of the model is motivated by several manifestations of bank-NBFI intermediation that we are trying to model. Consider, for example, a private credit firm that has made a loan to a mid-market firm. In a liquidity crisis, this mid-market firm may need support, e.g., temporary suspensions of debt payment obligations or an infusion of new loans. If the private credit firm can provide this support, through its own funds, through calls on existing investors, through raising new funds, or through drawing down on bank credit lines, then the mid-market firm survives. If the private credit firm cannot provide such support, however, the firm defaults on its loan, even though – had it survived the liquidity crisis – it may have ultimately been able to pay off the loan. Another example would be a mortgage servicer who does not have or cannot raise sufficient funds in a liquidity crisis to pay MBS investors while providing temporary loan modifications or forbearance to a homeowner. In such a case, the homeowner might be forced to default. And, as a final example, consider a physical commodity trading firm that, as part of its financing relationships with customers, supplies contingent liquidity for the purpose of making margin calls on commodity derivatives hedges. If this trading firm should itself experience stress in a liquidity crisis and are rendered either unable or unwilling to continue to provide contingent liquidity, its customers might default on those margin calls.

In the context of the model, in which a bank can always provide liquidity, it is a deadweight loss for a loan to default because the NBFI cannot provide liquidity. To mitigate this deadweight loss, the bank can offer a credit line to the NBFI, which is drawn down in a liquidity crisis to support the ultimate borrower. However, while the credit line does mitigate the deadweight loss, it raises the possibility that the NBFI might draw down and then default on its credit line. This could occur, in practice, because the ultimate borrower defaults on its advance from the NBFI or because the NBFI had allowed some of its funders to cash out and then failed to raise new funds. In any case, if the amount of the credit line equals the full size of the junior tranche, and if the default is also that full amount, then a default on the credit line is equivalent to the bank no longer having a first-loss, subordination buffer against the default of the loan.

To summarize the model to this point, junior claims are held by NBFIs so that banks, facing coarse capital requirements, can maximize their use of cheap, deposit funding. Banks offer credit lines to NBFIs to avoid the deadweight loss of the ultimate borrower defaulting on the loan, so long as the risk of extending these credit lines does not expose bank equity to even more significant losses.

The second stage of the model endogenizes the capital requirement, which enables the regulator to raise or lower the level of the capital requirement, but not to change its form, i.e., its sensitivity to subordination and credit lines, so as to eliminate the incentive for bank equity to have the NBFI hold the junior claim. The regulator faces uncertainty as to the magnitude of losses in the event that the loan defaults and cannot set state-contingent capital requirements. In other words, defaults might happen because of a minor, economic downturn or because of a great recession or depression, but the regulator may set only one capital requirement that will obtain through all eventualities. In making its decision, the regulator's objective function is to maximize economic activity, taken as some multiple of expected bank equity profits, minus the expected size of a bailout. Putting this all together, when the likelihood of catastrophic losses is low, the regulator sets relatively low capital requirements to encourage economic activity, knowing that the expected size of a bailout is relatively low. On the other hand, when the likelihood of catastrophic losses is high, the regulator sets relatively high capital requirements, sacrificing economic activity so as to keep the expected size of a bailout low.[1]

[1] The version of the model in this paper does not endogenize the extent of the bailout (fraction of bank deposits that are bailed out) or the trade-off between economic activity and the expected size of the bailout. These decisions can be thought of as happening at a higher level than the bank regulator, e.g., Congress rather than the Federal Reserve in the United States, but these components of policy can, in principle, be

It is also instructive to study the impact of bank credit lines to NBFIs on optimal capital requirements. We compare the optimal levels of capital requirements with and without credit-line provision allowed. The optimal level of capital requirements is generally higher when credit lines are available. The inclusion of credit lines transfers the credit risk in some states back to the bank balance sheets. To be precise, this happens in the model when bank credit lines are drawn down in liquidity risk states, but the NBFIs default on the drawn-down balances if the liquidity risk is severe. Therefore, the regulator sets a higher level of capital requirements in the presence of bank credit lines to reduce the expected bailout costs. This shows contingent liquidity provision by banks to NBFIs can have a first-order impact on the desirable level of bank capital requirements.

2 Related Literature

Recent work on nonbank financial intermediation has moved beyond the pre-crisis view of shadow banking of market-based credit creation as largely separate from bank-based credit creation. The central question is no longer only whether credit has migrated from banks to nonbanks, but how banks and nonbank financial intermediaries (NBFIs) are jointly organized. The literature offers three broad views. In the first, banks and NBFIs perform *parallel* activities: banks specialize in deposits, loans, payments, and access to public backstops, while NBFIs specialize in market-based intermediation. In the second, NBFIs *substitute* for banks when regulation or technology makes bank balance-sheet lending less attractive. In the third, which is our argument, bank and NBFI activities *transform* together: credit risk, funding, and liquidity support are reallocated across institutional boundaries without eliminating banks' central role in the intermediation chain.

A large theoretical literature studies the substitution view. Plantin (2015) shows that when banks can evade capital requirements through an NBFI sector, tighter bank capital regulation may increase NBFI activity and total risk on money-like liabilities. Huang (2018)

endogenized in the model as well.

similarly models an NBFI sector as off-balance-sheet financing used by banks for regulatory arbitrage. In his framework, implicit guarantees create an enforcement problem, and tighter bank regulation can increase the borrowing capacity of NBFIs by raising the value of regulatory arbitrage. Farhi and Tirole (2021) provide a broader normative theory in which prudential supervision, lender-of-last-resort access, deposit insurance, and core banking activities are complementary. An NBFI sector matters in their model because activities can migrate away from supervision and because cross-exposures can allow public liquidity to leak into the NBFI sector. Begenau and Landvoigt (2022) quantify related tradeoffs in a general equilibrium model with regulated banks and unregulated NBFIs. Tighter capital requirements increase NBFI intermediation in their calibration, but the overall system can become safer because reduced subsidies to commercial banks weaken NBFI incentives to take risk. These papers are important benchmarks for our analysis because they formalize how bank regulation changes the boundaries of intermediation. However, they generally emphasize migration, evasion, or competition rather than the specific contractual pattern in which banks retain senior funding and contingent liquidity exposures while NBFIs hold junior credit risk.

Empirical work confirms that regulation and balance-sheet constraints have indeed shifted lending toward nonbanks in recent decades. Irani, Iyer, Meisenzahl, and Peydro (2021) use supervisory syndicated-loan data and Basel III implementation shocks to show that less-capitalized banks reduce loan retention, especially for loans with higher capital requirements, and that nonbanks step in. They also find that loans funded by nonbanks with fragile liabilities were less likely to be rolled over and had greater price volatility during the 2008 crisis. Chernenko, Erel, and Prilmeier (2022) study direct nonbank lending and show that firms with negative EBITDA and high leverage are more likely to borrow from nonbanks, particularly where local banks are more constrained by strict supervision or leveraged-lending guidance. Jang (2026) adds that private credit lenders are not merely passive arm's-length investors: in loans to private-equity-backed firms, direct lenders use covenants extensively, renegotiate actively after violations, and often resemble banks in monitoring and restructuring. These papers help explain why riskier credit may be held outside banks. They do not, however, focus on the interplay with banks in the resulting intermediation chains.

The strongest evidence for continuing bank centrality comes from work on bank-NBFI funding and liquidity linkages. In our prior work, Acharya, Cetorelli, and Tuckman (2024, 2025), we argued that NBFI growth is better understood as "transformed intermediation" than as a clean migration of activity away from banks. Using financial accounts, regulatory data, case studies, and market-based evidence, we document that NBFIs are especially dependent on banks for funding and liquidity, including through senior loans, warehouse financing, credit lines, and derivatives-related liquidity support. The core transformation is asymmetric: NBFIs take on junior credit exposures and activities outside the bank regulatory perimeter, while banks retain senior exposures and contingent liquidity obligations. This evidence directly motivates our theoretical mechanism. It also distinguishes our paper from substitution models: the relevant institutional object is not a bank and an NBFI competing to make the same loan, but a joint financing structure in which each intermediary holds a different claim and performs a different liquidity function.

Several recent papers document this structure in particular markets, e.g., private credit[2] and mortgages.[3] Together, these papers support the transformation view: bank exit from

[2]In private credit, Chernenko, Ialenti, and Scharfstein (2024) challenge the simple claim that private credit grew because nonbanks are more levered or hold less capital. They show that business development companies are well capitalized under bank-style capital frameworks. Their more relevant point for our purposes is that, given capital treatment and funding costs, banks may prefer over-collateralized lending to BDCs and other private credit funds over direct middle-market lending. Banks thereby use their low-cost funding to hold safer claims on NBFIs, while private credit vehicles hold the riskier underlying corporate loans. This mechanism is close to ours: coarse capital requirements can make the senior claim on the NBFI more attractive to banks than the underlying credit exposure. In another related paper, Haque, Jang and Wang (2025) provide empirical evidence documenting the emergence of an elongated intermediation chain from banks to private credit stemming from varying funding costs as driven by monetary policy.

[3]Mortgage intermediation provides a parallel set of facts. Kim, Laufer, Pence, Stanton, and Wallace (2018) show that nonbank mortgage companies rely heavily on warehouse lines to fund originations before securitization and face liquidity risk from pipeline aging, rollover risk, covenant violations, and servicing advances. Benson, Kim, and Pence (2023) show that when large banks exited FHA mortgage aggregation, nonbank aggregators and vertically integrated originators entered, but the exiting banks continued to fund the nonbanks through warehouse credit. Jiang (2023) studies the same broad vertical structure from an industrial-organization perspective. She shows that nonbank mortgage originators are funded by banks that also compete with them, and that banks' upstream funding power can soften downstream mortgage competition.

one activity may coexist with bank funding of the NBFIs that replace them.

Credit lines are the most direct link between this evidence and our theory. Banks have a special role in providing contingent liquidity because deposits, deposit insurance, and central-bank backstops make them natural liquidity insurers. Kashyap, Rajan, and Stein (2002) provide the canonical explanation for the coexistence of deposit-taking and credit-line provision, and this logic is extended to NBFIs in recent work. Yankov (2020) documents that after the introduction of the Liquidity Coverage Ratio, banks increased liquid-asset holdings while NBFIs reduced liquid assets and increased reliance on bank credit lines. Acharya, Gopal, Jager, and Steffen (2024) show that bank credit lines to real-estate investment trusts (REITs) and business development companies (BDCs) transmit commercial-real-estate and mid-market firm risks back to banks: REITs and BDCs, due to institutional restrictions, have higher credit-line utilization than nonfinancial borrowers, draw more in stress, and create indirect exposures that are not captured by direct bank lending. Xu (2025) provides the closest theoretical complement, documenting that credit lines account for most bank funding to nonbank lenders and developing a model in which banks' liquidity advantage makes them natural providers of contingent liquidity. Xu emphasizes that endogenous credit limits and fees can discipline risk-taking and may improve welfare relative to cash or term loans. Our paper shares the focus on bank-provided contingent liquidity, but places it inside a broader claim-splitting arrangement: banks provide senior funding and liquidity support because NBFIs holding junior claims may otherwise be unable to continue funding or supporting loans in crisis states.[4]

Finally, the literature has important regulatory implications. Metrick and Tarullo (2022)

[4]Other work emphasizes bank-NBFI coexistence for different reasons. Donaldson, Piacentino, and Thakor (2021) show that banks' low funding costs can create a soft-budget-constraint problem, while nonbanks' higher funding costs make termination threats credible. Lyonnet and Chretien (2026) model traditional banks and NBFIs as complements because NBFIs rely on traditional banks to buy assets in crises, while traditional banks benefit from crisis discounts. Cetorelli, Cisternas, and Sarkar (2025) organize such mechanisms through a function-based view of intermediation in which banks and NBFIs may compete or cooperate depending on the function being performed. Cetorelli and Prazad (2024) show that even within bank holding companies, nonbank subsidiaries can create liquidity-management synergies through intercompany funding and implicit liquidity support. These papers are complementary to ours because they reject a static entity-based view of intermediation.

propose congruent regulation: activities posing similar systemic risks should be regulated with similar stringency regardless of institutional form. Farhi and Tirole (2021) similarly emphasize that public insurance and supervision must be designed together when activities can migrate. Our framework instead focuses on the regulator's problem when the contractual form itself is endogenous and capital rules are necessarily coarse. Regulators understand that lower capital requirements encourage economic activity but also encourage subordination and bank credit-line support to NBFIs. In benign states, this structure expands lending by allowing banks to deploy cheap funding against senior claims; in crisis states, it creates bailout risk because NBFIs may need bank or public liquidity to support the underlying loans. The distinctive implication is that bank capital requirements affect not only whether banks or NBFIs lend, but also how claims and liquidity obligations are split between them.

3 Model

There are four dates and four agents. At date 0, the regulator sets bank capital requirements. At date 1, a bank makes and funds a loan, possibly including a junior claim, and, if so, an NBFI raises funds to purchase that junior claim. At date 2, there is a possible realization of a liquidity shock, in which both the ultimate borrower and the NBFI may experience liquidity stress and, as a remedy, the NBFI draws down bank credit lines when available. Finally, at date 3, loan payoffs are realized and distributed according to the seniority of the capital structures. Also, at this point, the regulator may step in to bailout bank depositors.

The bank makes a unit loan, which either pays off, returning a rate $R_u > 0$, or defaults, returning a rate $R_d < 0$. The bank finances the loan with some combination of deposits, equity, and a "junior" or "subordinated" tranche with principal $s \in [0, 1]$. In addition, the bank may offer a credit line to the NBFI that holds the junior tranche. Contractually, losses are absorbed first by the junior tranche, then by equity, and then by deposits, but, as detailed below, an NBFI that defaults on its credit line will leave bank equity with the first loss. For simplicity, the model assumes that R_d is sufficiently negative to wipe out the junior claim and bank equity. Lastly, the government, which sets a policy with respect to bailing out depositors, also sets a minimum capital or equity funding level for the loan, $w(s, L)$, where $L \in [0, s]$ is the size of the credit line. This capital requirement naturally decreases in s, but increases in L. We furthermore assume that the bank will either not offer a credit line or will offer a credit line equal to the size of the junior tranche, which means that L equals either 0 or s. For now, the capital requirement, $w(s, L)$, is given exogenously, but is endogenized in the context of the regulator's optimization problem in Section 3.5. All agents are assumed to be risk neutral.

The capital structures in the model are summarized by the balance sheets of the bank and NBFI in Figure 4. The top panel shows the balance sheets without credit lines or with undrawn credit lines. The balance sheet on the left, a snapshot before the realization of the loan's return, shows that the bank asset is the unit loan, and its liabilities are the junior tranche, deposits, and bank equity. At the same time, the NBFIs assets are the junior tranche and its liabilities are its own equity.[5] The balance sheets on the right of the top panel is a snapshot of what happens after the loan defaults. The junior tranche is wiped out, so that its value – along with the equity value of the NBFI is 0. The bank's asset, namely the loan, is, as assumed, worth $1 + R_d$, which value is assigned first to the depositors, to the extent of their principal claim of $1 - s - w(s, L)$, and then the residual, $R_d + s + w(s, L)$, to bank equity.

The bottom panel show the corresponding balance sheets with the credit line drawn during a liquidity shock. To the left, before loan realization, the NBFI has funded a quantity of redemptions L from its equity holders by drawing down its credit line from the bank,

[5] An alternative setup, in which it would be natural for the NBFI to keep the residual profits, would be for the NBFI to make the loan and sell a senior tranche to the bank. The amount of deposit funding in this alternative setup is the same, namely, $1 - s - w(s, L)$, as is the post-default value of bank equity without credit lines ($R_d + s + w(s, L)$) and with credit lines ($R_d + s - L + w(s, L)$). Therefore, the incentives of the bank to subordinate and offer credit lines would remain unchanged, though its profits would, of course, be zero. In this setup, however, the regulators would have to use NBFI profits as an index of economic activity. This alternative setup is illustrated in a diagram of the balance sheets of banks and NBFIs in Figure A1 in the Appendix.

w/o Credit Line or w/ Undrawn Credit Line

Before Loan Realization

Bank		NBFI	
Assets	Liabilities	Assets	Liabilities
Loan: 1	Jr. Tranche: s	Jr. Tranche: s	Equity: s
	Deposits: $1 - s - w(s, L)$		
	Equity: $w(s, L)$		

Loan Default, Wiping Out Jr. Tranche

Bank		NBFI	
Assets	Liabilities	Assets	Liabilities
Loan: $1+R_d$	Jr. Tranche: 0	Jr. Tranche: 0	Equity: 0
	Deposits: $1 - s - w(s, L)$		
	Equity: $\boldsymbol{R_d + s + w(s, L)}$		

w/ Drawn Credit Line L ≤ s to Meet Redemptions of Limited Partners

Before Liquidity and Loan Realizations

Bank		NBFI	
Assets	Liabilities	Assets	Liabilities
Loan: 1	Jr. Tranche: s	Jr. Tranche: s	Equity: $s\text{-}L$
Credit Line: L	Deposits: $1 - s - w(s, L)$		Credit Line: L
	Fed Loan: L		
	Equity: $w(s, L)$		

NBFI Cannot Raise New Equity; Defaults on Credit Line; and Loan Defaults

Bank		NBFI	
Assets	Liabilities	Assets	Liabilities
Loan: $1+R_d$	Jr. Tranche: 0	Jr. Tranche: 0	Equity: 0
Credit Line: 0	Deposits: $1 - s - w(s, L)$		Credit Line: 0
	Fed Loan: L		
	Equity: $\boldsymbol{R_d + s - L + w(s, L)}$		

Figure 4: Bank Originates Loan and Securitizes it to the NBFI who Manages it

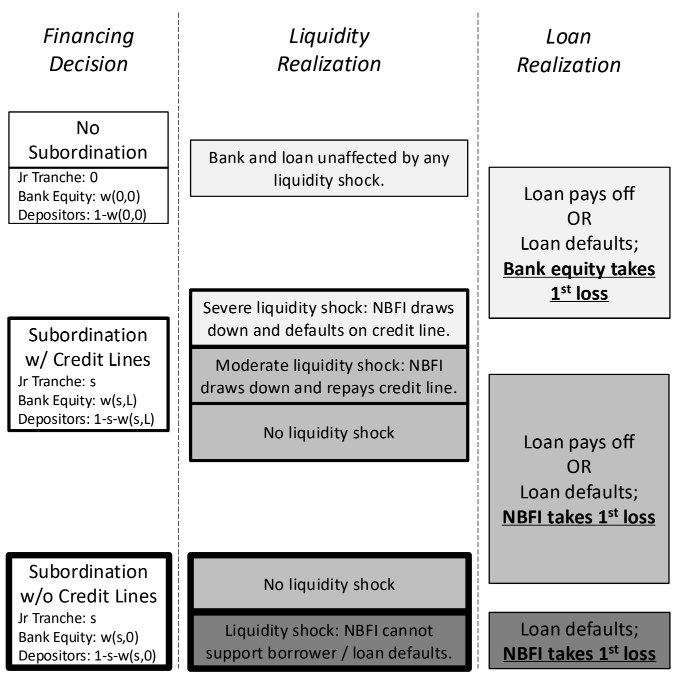
thereby reducing its equity liabilities by L and adding a credit line liability of L.[6] The bank, in turn, funds its extension of that credit line by borrowing from the Federal Reserve, thereby increasing its liabilities by that borrowing of L and increasing its assets by the credit line extension to the NBFI. To the right, after a loan default, the junior tranche is again worth 0, wiping out the value of remaining NBFI equity. The value of bank equity is the residual of the value of the loan, $1 + R_d$, from the principal claims of the depositors and the Federal Reserve, which comes to $R_d + s - L + w(s, L)$. Comparing this value of bank equity with the value of $R_d + s + w(s, L)$ in the case of no credit lines, given in the right of the top panel, illustrates an important feature of the model's setup: credit lines that are drawn and defaulted upon reduce the buffer of subordination one-to-one, that is, from s to $s - L$. In the special case we assume, namely $L = s$, the default on drawn credit lines completely wipes out the buffer of subordination.

[6]Another motivation for the NBFI to draw down its credit line, which is particularly important for the rest of the model's development, is to lend to the ultimate borrower through a liquidity shock.

3.1 Timing and State-Space

The timing and state-space of the model are described in the left panel of Figure 5. At the initiation of the loan, the bank makes its financing decision. Due to the cost advantage of deposit funding, detailed below, the bank always contributes the minimum required amount of equity capital. Therefore, the bank will choose among three financing cases: one, *No Subordination*, in which the bank funds the unit loan with $w(0,0)$ of equity and the remainder, $1-w(0,0)$, with deposits; two, *Subordination with Credit Lines*, in which the bank funds the unit loan with a subordinated tranche of s, combined with a credit line of L, $w(s,L)$ of equity, and the remainder, $1-s-w(s,L)$, with deposits; and three, *Subordination without Credit Lines*, in which the bank funds the unit loan with a subordinated tranche of s, $w(s,0)$ of equity, and $1-s-w(s,0)$ of deposits.

After the financing decision are the realizations of liquidity conditions and then of loan outcomes, shown in the second and third columns of the left panel of Figure 5.



Claim	Loan Pays Off	Loan defaults: Bank equity takes 1st loss	Loan defaults: NBFI takes 1st loss
Jr Tranche	$s(1+C)$	L	0
Bank Equity	$1+R_u-s(1+C)-(1-s-w(s,L))\cdot(1+\beta r_f-y+\theta)$	0	0
Depositors	$(1-s-w(s,L))\cdot(1+\beta r_f-y+\theta)$	$\phi(1-s-w(s,L))+(1-\phi)(1+R_d-L)$	$\phi(1-s-w(s,L))+(1-\phi)(1+R_d)$
Government	0	$\phi(R_d+s+w(s,L)-L)$	$\phi(R_d+s+w(s,L))$
Total	$1+R_u$	$1+R_d$	$1+R_d$

Figure 5: Timing, State-space and Payoff

In the case of *No Subordination*, liquidity conditions do not affect the bank or the loan, reflecting the model's emphasis on banks' special access to liquidity, both through their deposit franchises and through explicit and implicit government support. Therefore, in this case, as shown in the light-gray box in the *Loan Realization* column, the loan either pays off or defaults, and furthermore, because there is no subordinated claim, bank equity takes the first loss.

In the case of *Subordination with Credit Lines*, there are several combinations of liquidity and loan outcome realizations, reflected by the three rectangles under liquidity realization that branch into two rectangles under loan realization. The top two liquidity realizations depict a liquidity shock, in which the NBFI draws down on the credit line. In the top rectangle, a "severe liquidity" shock, the NBFI defaults on the credit line. Along this path, the matching loan realization is once again, the light-gray box: the loan either pays off or defaults, and furthermore, because the NBFI defaulted on the credit line of size equal to the junior tranche, there is no remaining subordination, and bank equity takes the first loss upon default. This feature of the model captures the severe liquidity and credit stresses in which defaults on credit lines to NBFIs impact banks and, by extension, depositors.

In the second of the liquidity realization rectangles, a "moderate" liquidity shock, the NBFI succeeds in repaying the credit line before the loan realization. Along this path, the matching loan realization is the medium-gray box: the loan either pays off or defaults and furthermore, because the NBFI has paid off the credit line, the junior tranche is intact, and the NBFI takes the first loss upon default.

In the third of rectangles of this case, there is no liquidity shock, and the medium-gray box obtains again: the loan either pays off or defaults, and, in the case of default, the NBFI absorbs the first loss.

In the third case, *Subordination without Credit Lines*, there are two possible liquidity realizations. In the first rectangle, there is no liquidity shock, and the loan realization is again described by the medium-gray box: the loan either pays off or defaults, and the NBFI takes the first loss upon default. In the second rectangle of this case, there is a liquidity shock. Without a credit line, however, the NBFI has no means of supporting the borrower and the loan always defaults. This loan realization is described in the dark-gray box in the loan realization column. This path generates the social benefit of credit lines: without liquidity support from banks, either directly (as in the case of *No Subordination*) or indirectly through NBFIs (as in the case of *Subordination with Credit Lines*), liquidity shocks trigger defaults on loans that might have paid off with liquidity support.[7]

For the mathematical development of the model, note the following feature of the state-space as portrayed in Figure 5. While there are many paths through liquidity and loan realizations, there are, in the end, only three possible outcomes: the loan pays off; the loan defaults, while bank equity takes the first loss; and the loan defaults, while the NBFI takes the first loss. The text now describes the payoffs to each claim in each of these three outcomes, which are recorded in the right panel of Figure 5.

3.2 Payoffs Across Claimants

In reality, either the bank or the NBFI might have market advantages and pricing power with respect to finding the borrower and arranging the loan, which would lead to some division of the profit accruing from the loan. For simplicity, here, however, it is assumed that the bank has all of the market power, which means that the (risk-neutral) NBFI and depositors are paid fair market returns for their claims, while any residual profit accrues to bank equity.

The junior tranche has a principal of s, which is optimized by the holders of bank equity. If the loan pays off, the tranche pays off its principal plus a spread or coupon rate, C, for a total of $s(1+C)$. The coupon C is determined by a fair-pricing condition so that the junior tranche, in expectation, earns the risk free rate, r_f. If the loan defaults, while the bank takes the first loss, which happens for $s > 0$ only if the NBFI defaults on the credit line, then the tranche actually preserves its principal value $L = s$. This corresponds economically to the case of the NBFI drawing down its credit line to pay off its capital providers and then

[7] This feature of the model is introduced in a particularly simple way. Even a very small amount of subordination without credit lines, combined with a liquidity shock, results in a default on the loan.

defaulting on that line.[8] On the other hand, if the loan defaults, while the NBFI takes the first loss, then the junior tranche is worthless.

Bank equity is the residual claimant in the event that the loan pays off, that is, it receives the proceeds from the loan, $1+R_u$, minus payments to the junior tranche and to depositors. Also, by assumption, as mentioned earlier, if the loan defaults, R_d is sufficiently negative so that bank equity is worthless.

Bank depositors are promised the principal of their deposits, $1-s-w(s,L)$, plus a return on those deposits of $\beta r_f - y + \theta$. The parameter $\beta < 1$ is the usual deposit beta, reflecting the empirical reality that deposit rates respond less than one-to-one to changes in the riskless rate. The parameter y is a convenience yield, which denotes the extent to which depositors accept a lower interest rate in exchange for the provision of bank services. And the parameter θ is the spread that depositors require so that, accounting for defaults on deposit liabilities, the expected return on deposits equals $\beta r_f - y$. Given this formulation, if the loan pays off, depositors receive their deposited amount plus promised interest, that is, $[1-s-w(s,L)]\cdot[1+\beta r_f - y + \theta]$. However, if the loan defaults, depositors receive a fraction ϕ of their principal at par and a fraction $1-\phi$ of their principal at the residual value of the bank, which is either $1+R_d < 1$, if the NBFI has taken the first loss, or $1+R_d-L$, if the NBFI had withdrawn and then defaulted on its credit line of L.

The last participant is the government, which sets a policy of bailing out depositors that is represented by the fraction ϕ: the government makes up any difference between the value of the bank after a default on the loan, $1+R_d$, and the claims paid out to the depositors – and also to the NBFI, in the case of an NBFI default on its credit line. As mentioned above, the model assumes for simplicity that R_d is sufficiently negative so that that government actually has to contribute funds in a bailout. Or, in terms of the entries in the right panel of Figure 5, the government cash flows in the two rightmost columns are always negative.

[8] In an alternative motivation, with the NBFI drawing on the credit line to lend to the ultimate borrower, a default on the loan and on the credit line would move L to some creditor of the ultimate borrower rather than to NBFI equity. However, the bank's subordination buffer would still have declined by L.

3.3 Probability Space

Figure 6, which diagrams the probability space of the model, is described in this subsection. As in Figure 5, the description is broken up into the three different financing choices of the bank. In terms of probabilities, the sum of outcome probabilities within each financing choice equals 1.

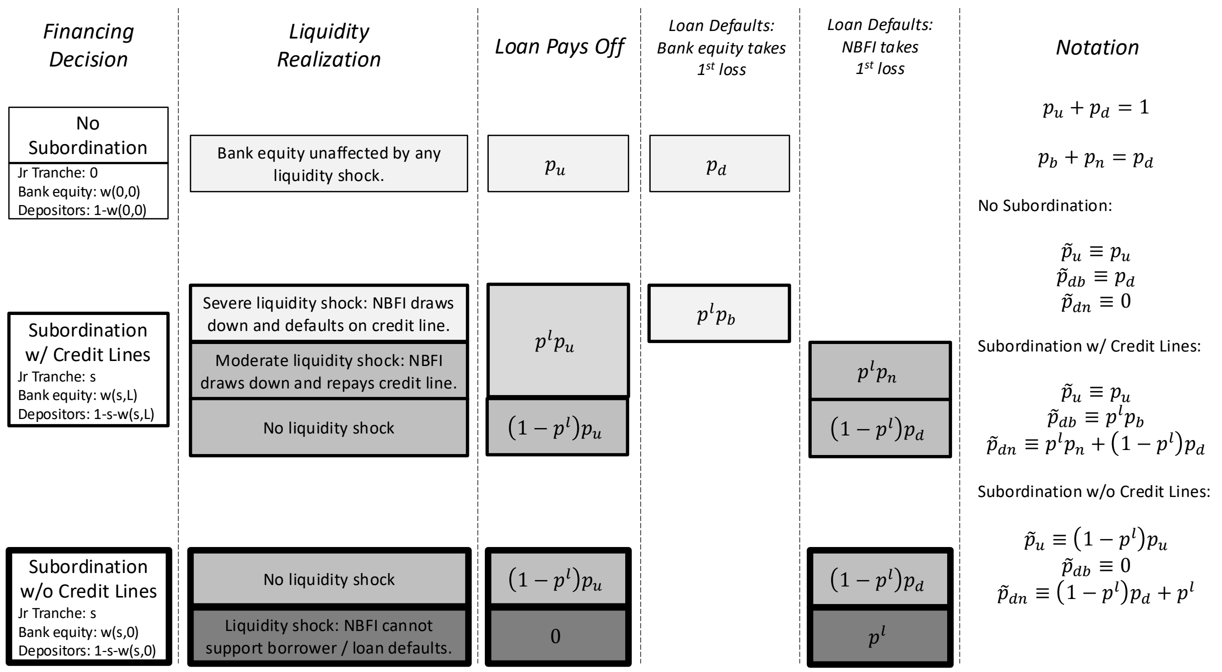


Figure 6: Probability Space

In the case of *No Subordination*, the probability that the loan pays off is p_u. The probability that the loan defaults is $p_d \equiv 1 - p_u$. In this case, according to the state-space of Figure 5, all of p_d corresponds to bank equity taking the first loss. Put differently, in this case, the probability that the loan defaults and that the NBFI takes the first loss is 0.

In the case of *Subordination with Credit Lines*, the probability of a liquidity shock is p^l and the probability of no liquidity shock is $1 - p^l$. If there is a liquidity shock, it can be followed by the loan paying off, with probability p_u, or by the loan defaulting, with probability p_d. Here, however, consistent with Figure 5, p_d is further broken up into two possibilities: the severe liquidity shock in which the NBFI itself defaults on the credit line, with probability p_b, and the moderate liquidity shock in which the NBFI repays the credit line but the underlying loan defaults, with probability p_n. Note that $p_b + p_n = p_d$, so that the sum of the probabilities of the three possibilities after a liquidity shock is p_l, that is, $p^l p_u + p^l p_b + p^l p_n = p^l$. If there is no liquidity shock in this financing case, then – again consistent with Figure 5 – there are only two possibilities: the loan pays off or the loan defaults with the NBFI taking the first loss. The overall probabilities of these two outcomes are $(1 - p^l)p_u$ and $(1 - p^l)p_d$, which, of course, sum to $1 - p^l$.

In the last case, *Subordination without Credit Lines*, there is either no liquidity shock, with probability $1 - p^l$, or a liquidity shock, with probability p^l. Consistent with Figure 5, if there is no liquidity shock, the loan either pays off or defaults, with the NBFI taking the first loss. The overall probabilities of these two outcomes are $(1 - p_l)p_u$ and $(1 - p_l)p_d$, respectively, which sum to $1 - p^l$. If there is a liquidity shock in this case, then, again consistent with Figure 5, the loan always defaults: without any liquidity support, the NBFI cannot support the underlying borrower. The probability of this outcome, therefore, is the full p^l. Note that this last outcome is the model's mechanism for creating a deadweight loss in the absence of credit lines.

The rightmost column of Figure 6 describes some additional notation that will be used in working through the model. More specifically, $\tilde{p}_u$, $\tilde{p}_{db}$, and $\tilde{p}_{dn}$ are defined case by case to be the probability that the loan pays off, the probability that the loan defaults with bank equity taking the first loss, and the probability that the loan defaults with the NBFI taking the first loss. In each case, the sum of these three probabilities is 1.

3.4 Bank Equity Optimization

Bank equity maximizes its expected profit, $\pi(s, L)$, with respect to subordination to NBFIs, s and credit-line provision to the NBFI, L, with payoffs given in the right panel of Figure 5 and probabilities in Figure 6:

$$\max_{s,L} \pi(s, L) = \tilde{p}_u[1 + R_u - s(1 + C) - (1 - s - w(s, L))(1 + \beta r_f - y + \theta)] \tag{1}$$

However, the deposit and junior tranche spreads, θ and C, respectively, are subject to fair pricing conditions. More specifically, depositors have to earn an expected return of $\beta r_f - y$ on their $1 - s - w(s)$ of deposits, and the NBFI has to earn an expected return of r_f on its investment of s. Mathematically, θ must satisfy

$$
\begin{aligned}
&\tilde{p}_u(1 - s - w(s,L))(1 + \beta r_f - y + \theta) \\
&+ \tilde{p}_{db}[\phi(1 - s - w(s,L)) + (1-\phi)(1 + R_d - L)] \\
&+ \tilde{p}_{dn}[\phi(1 - s - w(s,L)) + (1-\phi)(1 + R_d)] \\
&= (1 - s - w(s,L))(1 + \beta r_f - y)
\end{aligned} \tag{2}
$$

and C must satisfy

$$
\tilde{p}_u s(1 + C) + \tilde{p}_{db} L = s(1 + r_f) \tag{3}
$$

Substituting expressions for θ and C from (2) and (3), respectively, into (1) gives the following expression for the optimization of bank equity:

$$
\begin{aligned}
\max_{s,L} \pi(s,L) = \; &\tilde{p}_u R_u + (1-\phi)R_d(\tilde{p}_{db} + \tilde{p}_{dn}) \\
&+ \tilde{p}_{db}\phi L - (\beta r_f - y) \\
&- (s + w(s,L))[\phi(\tilde{p}_{db} + \tilde{p}_{dn}) + r_f(1-\beta) + y]
\end{aligned} \tag{4}
$$

3.4.1 A Special Case of $w(s,L)$ and Parameters for Examples

In general, the functional form of $w(s,L)$ would have the following properties:

- $1 > w(0,L) > 0$: With no subordination, required equity capital is in $(0\%, 100\%)$;
- $w(s,0) \leq 1 - s$: Without a credit line, required equity capital does not exceed the loan amount;
- $w(1,0) = 0$: Without a credit line, no bank equity capital is required if the junior tranche finances the entire loan;
- $w(s,L) \geq w(s,0)$: For a given subordination, required equity capital is no less than

without a credit line;

- $w_s(s, L) < 0:$ Required capital decreases in the amount of subordination; and
- $w_{ss}(s, L) \geq 0:$ Required capital is convex in the amount of subordination.

However, to prove the propositions presented in subsequent subsections and to present numerical examples, the capital requirement is specialized to take the form

$$w(s, L) = w_0[e^{-as} - (1 - L)e^{-a}] \tag{5}$$

With this functional form, the level of the capital requirement can be described by the single parameter w_0, which, for convenience, we restrict to the range $[\frac{1}{a}, 1]$. The parameter a, by contrast, describes how subordination and credit lines decrease and increase required capital, respectively. Note that, under the assumed functional form the effects of subordination and credit lines on the capital requirement do not depend on each other. In practice, this assumption does not seem unrealistic. Under Basel III standards, capital charges on bank credit lines to NBFIs are only based on the underlying credit quality of the obligors (a coarse distinction between investment-grade vs below investment-grade categories) and a fixed credit conversion factor of 40 percent, applicable to all commitments (BCBS, 2017).[9]

Furthermore, in the numerical examples, the parameters used are as in Table 1:

Table 1: Parameters for Numerical Examples

Variable	Value
p_u	0.7
p_b	0.1
p_n	0.2
R_d	-80%
$p_u R_u + p_d R_d$	8%
r_f	4%
β	0.35
y	0.4%
a	3

[9]In future drafts, we will add results on robustness to the specific functional form by considering alternative specifications.

The remaining parameters, w_0, p^l, and ϕ will be varied, as described below.

3.4.2 Model Results: Subordination Without Credit Lines

Starting the analysis of the model with the role of subordination, this subsection assumes that the bank does not offer credit lines, that is, $L = 0$. This assumption is relaxed in the next subsection.

The motivation of bank equity for subordination to the NBFI, that is, for selling the junior tranche, is evident from inspection of (3.4). Maximizing expected bank equity profits is related to minimizing $s + w(s, 0)$, or maximizing $1 - s - w(s, 0)$, which is the amount of deposit funding. Intuitively, because deposit funding is relatively cheap for banks, as captured by the parameters β and y, bank equity wants to rely on deposit funding as much as possible, but is constrained by the capital requirement, which is a decreasing function of subordination. Hence, increased subordination that increases deposit funding tends to increase expected profits.

There is another feature of the model, however, that complicates bank equity's use of subordination. As shown in Figure 5 and Figure 6, in the case of no subordination, and also in the case of subordination without credit lines and no liquidity shock, the loan can pay off or default. However, in the case of subordination without credit lines and a liquidity shock, the loan always defaults. This deadweight loss in the expected loan payoff due to the issuance of the junior tranche has two implications. First, subordination reduces the expected payoff on the loan, that is, the amount available to be split across all claimants, by an amount that increases with the probability of the liquidity shock, $\tilde{p}^l$. Second, from the perspective of bank equity, this deadweight loss is mitigated to the extent that the government bails out depositors. More mechanically, the greater the bailout, the less depositors need additional spread to compensate for possible loan defaults, and the greater the expected profit of bank equity.

The following propositions, proven in the appendix, formalize the discussion to this point.

Proposition 1 *When $L = 0$ and $\phi = 0$, there exists a $\bar{p}^l \in (0, 1)$ such that bank equity chooses an optimal $\hat{s} > 0$ for any $p^l \leq \bar{p}^l$. When $L = 0$ and $\phi > 0$, there exists a $\bar{\bar{p}}^l \in (0, 1)$, with $\bar{\bar{p}}^l > \bar{p}^l$, such that bank equity chooses an optimal $\hat{s} > 0$ for any $p^l \leq \bar{\bar{p}}^l$.*

Proposition 2 *When the optimal $\hat{s} > 0$, $\hat{s}$ is nondecreasing in w_0 and does not depend on the bailout policy ϕ.*

In words, Proposition 1 says that, without credit lines or bailouts, bank equity optimally chooses to subordinate so long as the probability of a liquidity shock is not too high. And with bailouts, bank equity chooses to subordinate at even higher probabilities of a liquidity shock. Proposition 2 says that, given that bank equity optimally chooses some positive amount of subordination, that amount stays the same or increases as the capital requirement increases. Put another way, the posited functional form of the capital requirement enables bank equity to sidestep higher capital requirements, at least to some extent, by increasing subordination and, thus, increase deposit funding. In addition, taken together, Propositions 1 and 2 say that, without credit lines, a bailout makes it more likely that bank equity optimally subordinates, but that the amount of the bailout does not increase the amount of subordination. Instead, as will be shown by example below, larger bailouts simply increase the profitability of bank equity.

The four panels of Figure 7 and Figure 8 numerically illustrate Propositions 1 and 2 using the parameters given in the previous subsection. Panels A and B of Figure 7 show the amount of deposit funding and the expected profitability of bank equity as a function of the amount of subordination. These two graphs fix some additional parameters: $p^l = 0$, $w_0 = 1$, and ϕ equals either 0 or 0.8. Panel A shows that the amount of deposit funding has a maximum at a subordination of $\hat{s} = 0.366$, which is, in fact, the optimal subordination chosen by bank equity, with or without bailouts. Panel B shows that bank profitability, again, with or without bailouts, has a maximum at that same $\hat{s}$. However, profitability is always at least as high with bailouts. More specifically, at very low levels and very high levels of subordination, the amount of deposit funding is very low, and there are no bailouts.

But for intermediate levels of subordination, expected bailouts increase the profitability of bank equity through the reduction of the deposit spread θ.

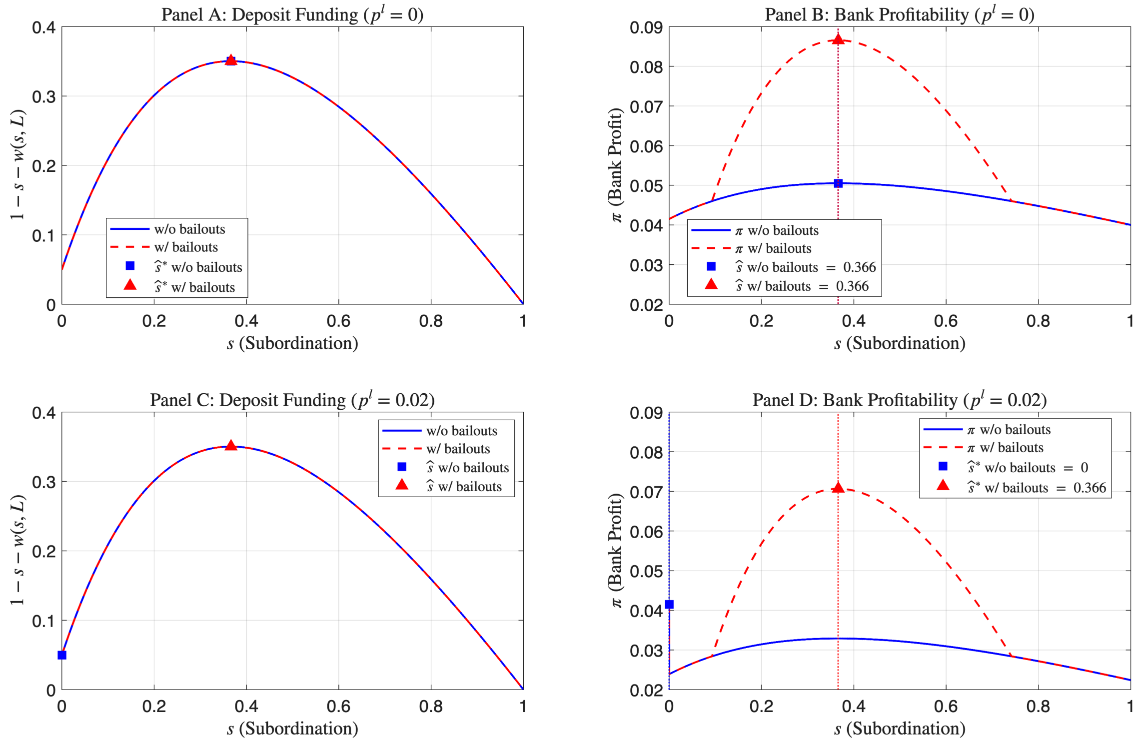


Figure 7: Deposit Funding and Bank Profitability vs. Subordination

Panels C and D of Figure 7 repeat the previous two panels, but with a positive probability of a liquidity shock, $p^l = 0.02$. The deadweight loss of the defaulting loan in the event of that liquidity shock reduces bank profitability for any amount of subordination. Without bailouts, i.e., $\phi = 0$, this deadweight loss (resulting in a higher deposit spread θ) outweighs the capital efficiency of subordination, and bank equity chooses not to sell a junior tranche, i.e., $\hat{s} = 0$. With bailouts, which limit the need for a higher deposit spread, optimal subordination is as before, $\hat{s} = 0.366$. However, bank profitability is still reduced because the higher p^l increases the probability of the loan's default.

Figure 8 graphs the optimal $\hat{s}$ as a function of the level of the capital requirement, w_0. Generally speaking, as discussed above, bank equity offsets increases in the level of capital requirements by increasing subordination and, therefore, increasing deposit funding. Figure 8, however, reveals some additional features of the model. For relatively low levels of

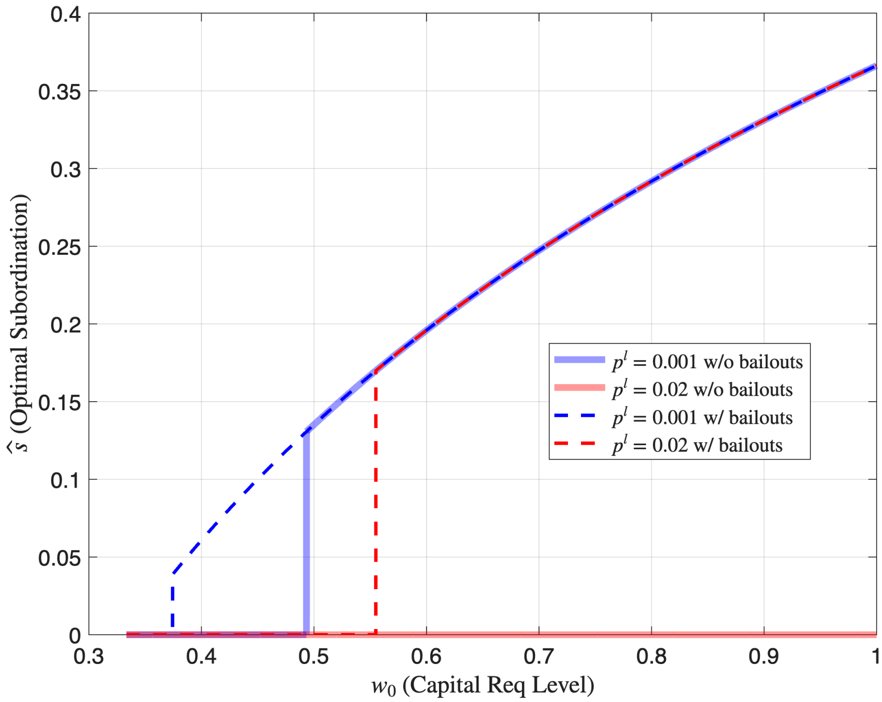


Figure 8: Optimal Subordination Level as a Function of the Capital Requirement

capital requirements, no subordination is necessary to maximize deposit funding. As w_0 increases, however, subordination becomes optimal at a point determined by the other parameters. For the low $p^l = 0.001$ case with bailouts, the deadweight loss from subordination is relatively small, and subordination jumps up from 0 relatively quickly. Subordination jumps next from 0 for the same low $p^l = 0.001$, but without bailouts. Next is the higher $p^l = 0.02$ with bailouts. Finally, for this higher $p^l = 0.02$ without bailouts, the deadweight loss of subordination through a liquidity shock is too large for bank equity to choose any level of subordination.

3.4.3 Model Results: Subordination With Credit Lines

As discussed above, a key feature of the model is that bank credit lines can offset the deadweight loss created by subordination by allowing an NBFI to help the underlying borrower to survive a liquidity crisis. Proposition 3 first shows that, without bailouts, credit lines are only optimally offered by bank equity if the probability of a liquidity crisis is large enough.

Proposition 3 *When $\phi = 0$, there exists a $\bar{p}^{lc} < \bar{p}^{l}$, such that it is optimal for bank equity to subordinate and provide a credit line to the NBFI $\hat{L} > 0$ if $p^l > \bar{p}^{lc}$ and $w_0 > \frac{1}{a-e^{-a}}$.*

Figure 9 illustrates this proposition for a low $p^l = 0$. Because there can be no liquidity crisis, there is no deadweight loss to be mitigated, and credit lines just increase capital requirements. Hence, Panel A of Figure 9 shows that deposit funding is lower with credit lines, and Panel B shows that bank equity profitability is lower with credit lines. Hence, bank equity chooses not to offer a credit line. Consistent with Proposition 1, however, there is an optimal amount of subordination, which is the same as in Figure 7.

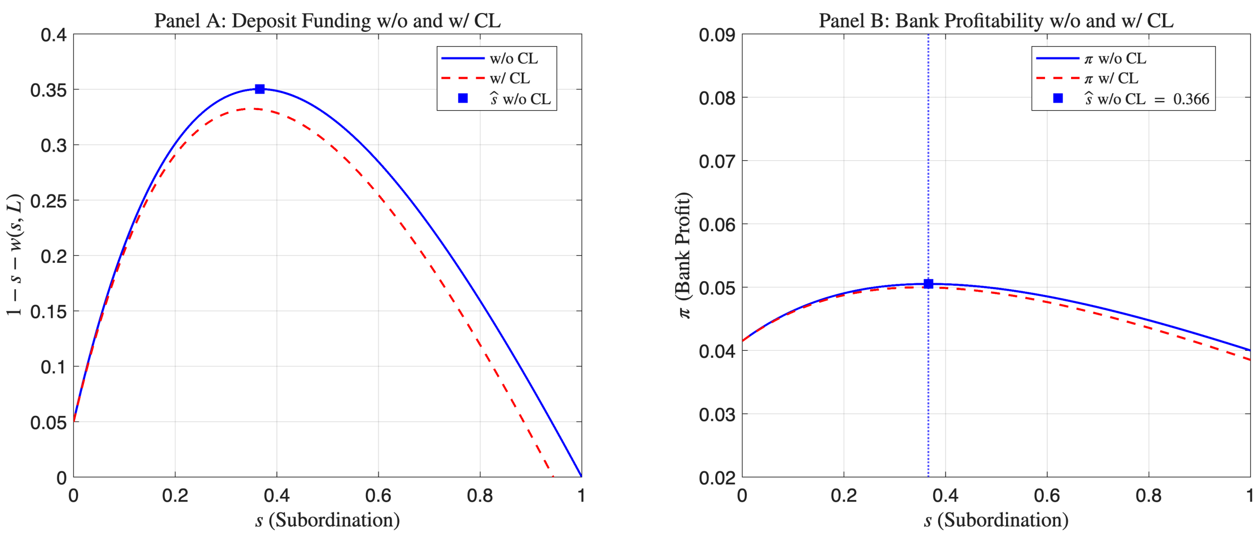


Figure 9: Deposit Funding and Bank Profitability vs. Subordination with and without Credit Lines

Figure 10 illustrates Proposition 3 for a relatively high $p^l = 0.02$. With this higher probability of a liquidity shock, a credit line reduces the deadweight loss of subordination and profitability is higher with credit lines for all levels of subordination. Bank equity optimally sells a junior tranche with $\hat{s} = 0.35$ and provides a credit line in that amount. Were the bank for some reason not allowed to offer a credit line, the deadweight loss of subordination would be too large and the optimal subordination is 0.

Although there is no related proposition, Figure 11 shows the impact of bailouts on profitability with subordination and credit lines. As in Figure 7, Panels B and D, for small

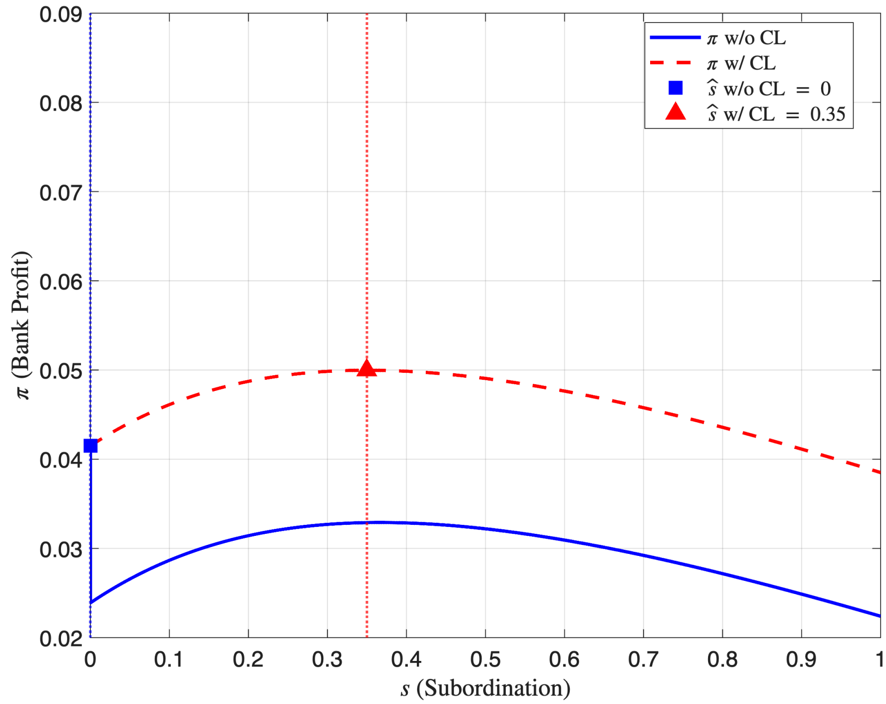


Figure 10: Bank Profitability vs. Subordination with and without Credit Lines in the Presence of Liquidity Shock

and large levels of subordination, the amount of deposit funding is small and bailouts do not come into play. However, for intermediate levels of subordination, bailouts flow directly into bank profitability through a reduced spread on deposits.

The next proposition describes how the level of the capital requirement impacts the provision of credit lines. The basic idea is like that of Proposition 2: as the level of capital requirements increase, bank equity seeks to increase the amount of deposit funding by subordinating. Here, however, bank equity may willingly incur the extra capital cost of credit lines to mitigate the deadweight loss of subordination.

Proposition 4 *The optimal credit line, $\hat{L}$, is nondecreasing in w_0 when $p^l > \bar{p}^{lc}$.*

Panels A and B of Figure 12 elaborate on this result numerically, with Panel A graphing the optimal subordination and Panel B graphing the optimal credit line, both against the level of the capital requirement, w_0. The amounts of optimal subordination without credit lines in Panel A are the same as the "with bailout" curves in Figure 8, and the corresponding

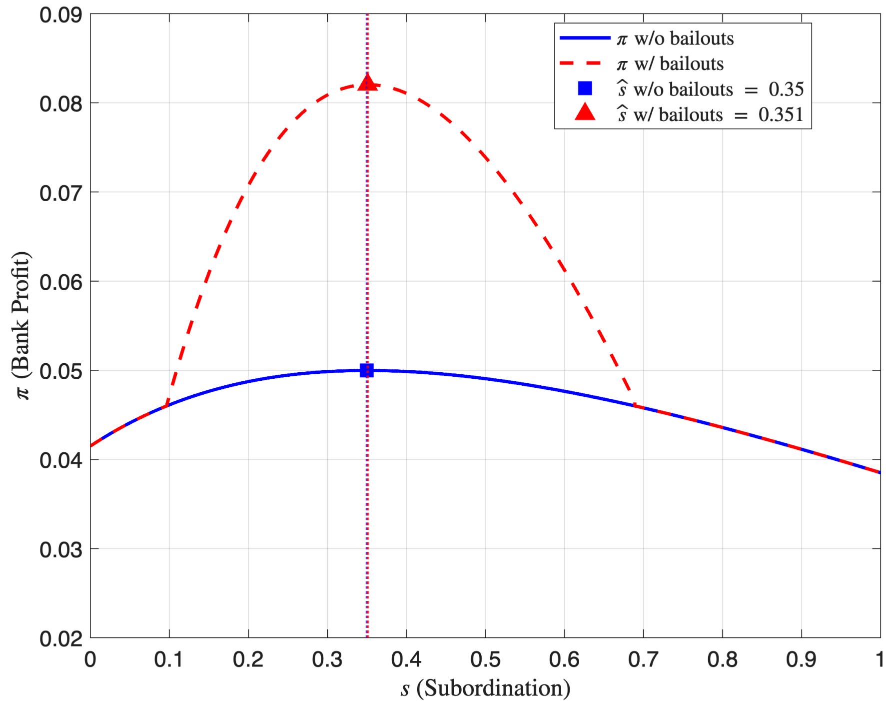


Figure 11: Bank Profitability vs. Subordination with Credit Lines with and without Bailouts

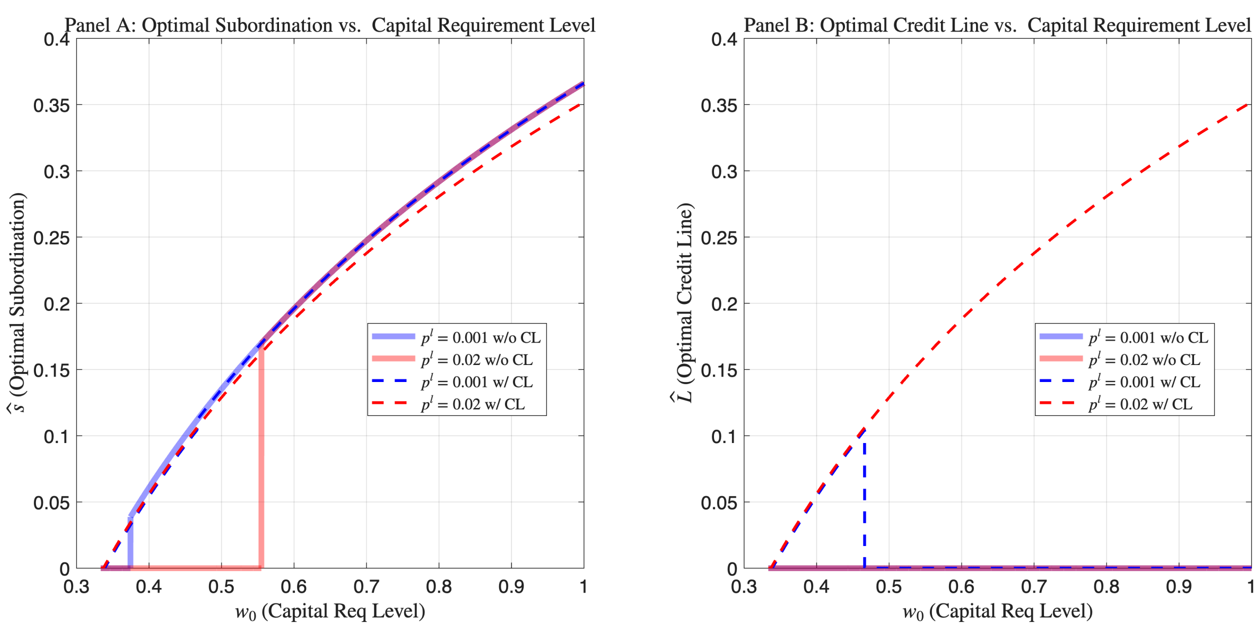


Figure 12: Optimal Subordination and Credit Line vs. Capital Requirement Level

optimal credit lines in Panel B are, by definition, always 0. With credit lines, Panel A shows that, once subordination is optimal, it increases with w_0, for both $p^l = 0.001$ and $p^l = 0.02$. Panel B shows that the optimal credit line increases with w_0, along with subordination, for the higher $p^l = 0.02$, as mitigating the deadweight loss of subordination is worth incurring the extra capital requirement of the credit line. For the lower $p^l = 0.001$, however, the optimal credit line increases with w_0 and subordination only for relatively low levels of w_0. At higher w_0, the higher capital requirement of the credit line outweighs overcoming the deadweight loss, and bank equity no longer finds it optimal to offer a credit line. Returning to Panel A, the optimal level of subordination is slightly lower for the higher p^l. In other words, the greater deadweight loss of subordination from a higher p^l lowers the optimal subordination, despite the credit line offered to mitigate that deadweight cost.

To summarize the results so far, subordination to NBFIs is a manifestation or mirror image of deposit-seeking by banks. As such, advantages to deposit-raising, e.g., convenience yield on deposits and access to safety nets, increase the bank-NBFI transformation whereby credit risks migrate to NBFIs away from banks as a regulatory arbitrage of (coarse) capital requirements. Furthermore, provision of credit lines by banks to NBFIs addresses the inefficiency from regulatory arbitrage as banks can manage contingent liquidity needs better than NBFIs. However, this can transfer credit risk back to banks (as shown in the bottom right panel of Figure 4), i.e., on-balance-sheet subordination to NBFIs may not reflect true or effective subordination in states where NBFIs draw down on bank credit lines. This can fundamentally alter the regulatory tradeoff in the design of optimal capital requirements, which we study next.

3.5 The Regulator's Optimization Problem

This subsection endogenizes the level of the capital requirement, w_0, which is one of the parameters in the functional form $w(s, L)$, given in Equation (5). As discussed below, changing this parameter allows the regulator to implement desired trade-offs between economic activity and expected bailouts. The other parameter of $w(s, L)$, namely, a, which governs the rate at which required capital falls with subordination and rises with credit lines, remains exogenous. The spirit of the model specifically requires that the regulator not be able to refine capital requirements for all assets under all combinations of subordination and credit lines. This regulatory wedge or imperfection is exactly what is posited as the reason banks can and do optimize over subordination and credit lines. A byproduct of this optimization exercise is to confirm that there is a robust set of parameters over which bank capital requirements are set at levels consistent with observing some subordination to NBFIs in equilibrium.

The regulator's problem is modeled as follows. While the bank knows the possible loan outcomes, R_u and R_d, before making its financing decision, the regulator must set capital requirements knowing only that there are two possible pairs of loan outcomes, only one of which will be realized before the bank's decision. These two pairs are denoted R_{u1} and R_{d1}, which obtain with probability P_1, and R_{u2} and R_{d2}, which obtain with probability $P_2 = 1 - P_1$. The expected return of the loan, however, is assumed constant across these two outcomes, that is, $p_u R_{u1} + p_d R_{d1} = p_u R_{u2} + p_d R_{d2}$. The first pair of loan outcomes is intended to reflect a more usual state of financial outcomes, in which defaults are relatively manageable, while the second pair is intended to reflect a great recession or depression scenario, in which defaults are relatively extreme. The regulator, as to be discussed, values both economic activity and avoiding expensive bailouts. If the regulator knew the realization of the loan outcomes before setting capital requirements, it would set capital requirements relatively low for outcome pair 1, in which bailouts are less expensive, and relatively high for outcome pair 2, in which bailouts are more expensive. But the regulator does not know which pair will obtain, which drives its optimization over w_0.

The regulator's objective function is the sum of two components. One, a parameter, δ, times expected bank profits, which are taken as a measure of economic activity, and two, the (negative) amount of the expected bailouts. The parameter, δ, is exogenous, reflecting instructions to the regulator from some higher authority in government, e.g., Congress in the United States. Mathematically, then, the regulator's optimization problem is written as follows:

$$
\begin{aligned}
&\max_{w_0} \pi^R(\hat{s}(\hat{L}, w_0), \hat{L}(\hat{s}, w_0)) \\
&= P_1[\delta\pi_1(\hat{s}, \hat{L}) + \phi\tilde{p}_d(R_{d1} + w(\hat{s}, \hat{L}) + \hat{s}) - \phi\tilde{p}_{db}\hat{L}] \\
&+ (1 - P_1)[\delta\pi_2(\hat{s}, \hat{L}) + \phi\tilde{p}_d(R_{d2} + w(\hat{s}, \hat{L}) + \hat{s}) - \phi\tilde{p}_{db}\hat{L}] \qquad (6)
\end{aligned}
$$

where, $\tilde{p}_d \equiv \tilde{p}_{db} + \tilde{p}_{dn}$, where $\hat{s}$ and $\hat{L}$ are respectively the bank's choice of subordination and credit-line provision to the NBFI. Note that we have suppressed their dependence on the state of the world for sake of parsimony.

For the purposes of numerical examples, the parameters in Table 2 are added or substituted for the parameters given in Table 1, unless specified otherwise in a particular figure:

Table 2: Parameters for Numerical Examples: Regulator's Problem

Variable	Value
P_1	0.6
R_{d1}	-70%
R_{d2}	-90%
p^l	0.10
ϕ	0.8
δ	0.6

3.5.1 The Effect of Weighting Growth Over Bailouts

The first result formalizes the intuition that the regulator will set capital requirements high if its weight on economic activity (i.e., bank profits) is low relative to its weight on avoiding bailouts, and will set capital requirements low if its weight on economic activity is high relative to its weight on avoiding bailouts.

Proposition 5 *If $p^l > \bar{p}^{lc}$, then there exists a $\bar{\delta}$ and a $\bar{\bar{\delta}}$ such that the optimal $w_0^* = \frac{1}{a}$ when $\delta > \bar{\delta}$ and the optimal $w_0^* = 1$ when $\delta < \bar{\bar{\delta}}$.*

Figure 13 illustrates Proposition 5. For low values of δ, the regulator optimally chooses a relatively high w_0 to reduce expected bailouts. When δ increases past a particular threshold, however, encouraging economic activity dominates reducing expected bailouts, and the optimal w_0 drops to a lower level.

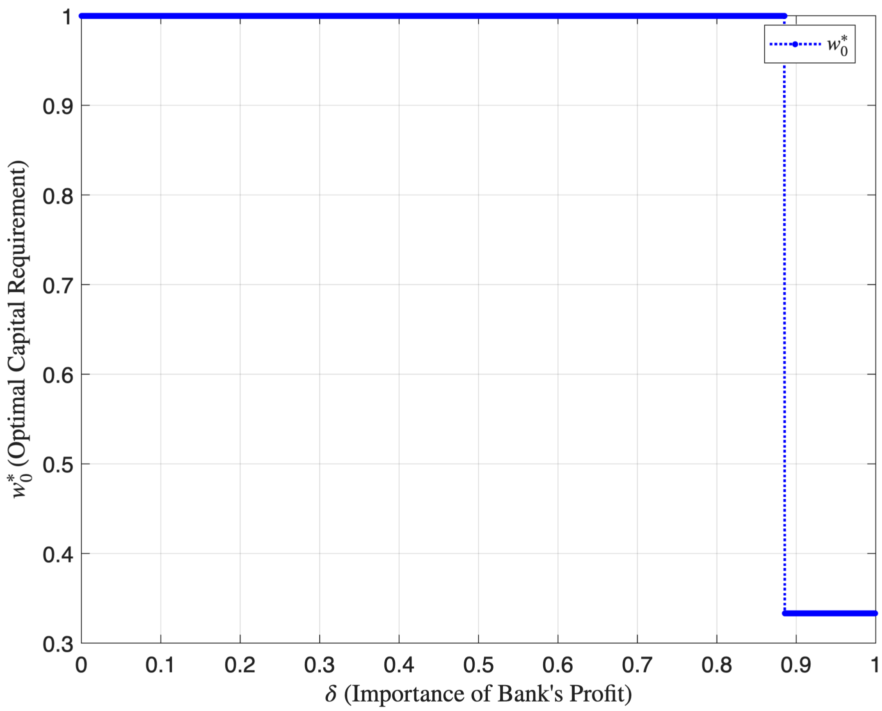


Figure 13: Optimal Capital Requirement as a Function of the Regulator's Weight on Bank Profits

Panels A through E of Figure A2 show how the regulator's choice of w_0 in Figure 13 play out in terms of other variables of interest. For low values of δ, when the regulator chooses a high w_0, the bank somewhat circumvents the higher capital requirements with higher subordination (Panel A), but is left with lower amounts of deposit funding (Panel B), and lower expected profits (Panel C). Also, at this high w_0, expected bailout costs are low (Panel D). By contrast, for high values of δ, when the regulator chooses a low w_0, the bank chooses a relatively low level of subordination, deposit funding is higher, expected bank profits are higher, and expected bailout costs are greater. Panel E shows the regulator's optimized expected objective function for varying values of δ. This objective increases in value with δ itself, as Panels C and D show that expected bank profits and expected bailout costs are constant over each of the regions corresponding to the two optimal levels of w_0. The bank's response to the reduction of the level of capital requirements by choosing a low level of subordination leads to the discontinuity in Panels A through D at the value of δ that first causes the regulator to drop the level of capital requirements. At that δ, the slope of the regulator's objective function increases, as the higher level of δ-weighted expected bank profits outweigh the greater expected bailout costs.

The optimal levels of capital requirements with and without credit-line provision allowed are illustrated in Figure 14 with $p^l = 0.01$. The optimal capital requirements with credit-line provision allowed are generally higher than those without credit-line provision allowed. In the region where the regulator first lowers capital requirements when credit-line provision is not allowed[10], capital requirements remain relatively high when credit-line provision is available. This is because credit-line provision transfers the credit risk back to bank's balance sheets when credit lines are drawn down and the NBFI defaults on the drawn-down balances in severe liquidity shocks. In other words, the drawn-down credit lines in bad-loan-outcome state, i.e., a great recession or depression scenario in which R_d is extremely low, result in considerably larger bailout costs than those in normal loan outcome. This is shown in Panels A and B in Figure A3 in the appendix. Given the uncertainty regarding loan outcomes and potentially large bailout costs in the bad-loan-outcome state, the regulator sets a higher level of capital requirements with bank credit lines to NBFIs in order to reduce expected bailout costs (Panel E in Figure A4).

3.5.2 The Effect of the Probability of Default

This subsection presents a series of graphs similar to those of the previous subsection, but instead of varying δ, varies the probability that the loan defaults, p_d. For these comparative statics, the ratio of the probabilities of the NBFI first loss and the bank first loss, p_n/p_b, is

[10] As the importance of bank profits (δ) increases in the regulator's objective, the regulator lowers capital requirements from a high level first to an intermediate level and only thereafter to the minimum level. This intermediate level is chosen to encourage deposit funding while still limiting expected bailout costs.

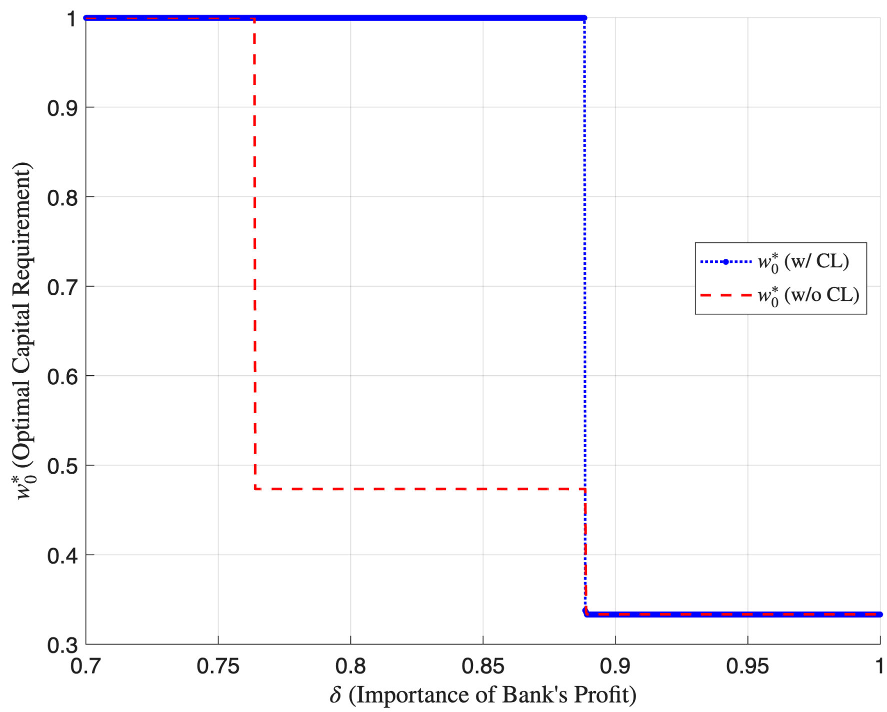


Figure 14: Optimal Capital Requirement with and without Credit-Line Provision Allowed

kept constant, as is the expected return on the loan, $p_u R_u + p_d R_d$. The focus here, therefore, is on varying the likelihood of default and bailout, while keeping other aspects of the model constant.

Figure 15 graphs the regulator's optimal choice of w_0 for different values of p_d. For small probabilities of default, when the expected size of bailout are low, capital requirements can be set low so as to encourage economic activity. However, as p_d rises above a certain level, the expected size of bailout dominate the considerations of economic activity and capital requirements are set at a higher level.

Panels A through E of Figure A5 show the effect of other model quantities as p_d varies, which all exhibit the same change at a critical level of p_d as in Figure 15. In panel A, when p_d and capital requirements are low, optimal subordination and credit lines are zero. However, for higher p_d and, therefore, capital requirements, the bank chooses a positive amount of subordination and credit lines to mitigate the resulting decrease in deposit funding. Nevertheless, as shown in Panel B, the amount of deposit funding is lower for higher levels

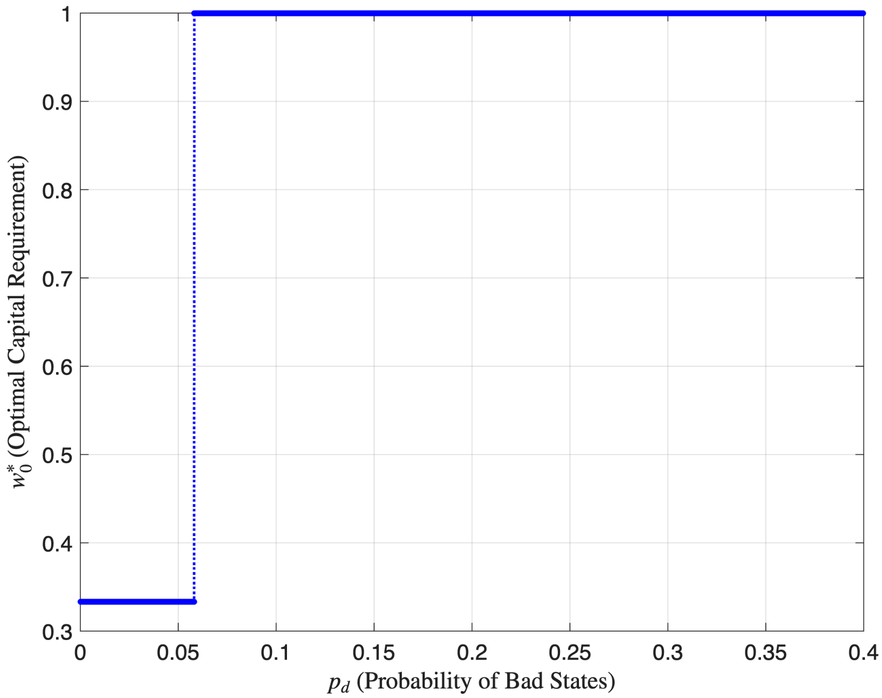


Figure 15: Optimal Capital Requirement as a Function of the Probability of Bad States

of p_d and capital requirements, as intended by the regulator, reduce the expected cost of bailouts. Panels C and D show that within each of the low p_d – low capital requirement regions, expected bank profits and government bailouts increase with p_d. Both downward jumps in Panels C and D occur at the level of p_d where the regulator switches from a low level of capital requirements to a high level, inducing an increase in bank's subordination choice. For a fixed w_0, the combination of a fixed expected return on the loan, a higher probability of default, and protection of depositors through bailouts results in a lower spread to depositors. Furthermore, the credit line to the NBFI in a severe liquidity crisis reduces the spread to the junior tranche. Lastly, Panel E shows that the regulator's objective falls with p_d, as the expected cost of the bailout increases, although the fall is slower in the region with higher capital requirements.

3.5.3 The Effect of the Extent of Bailouts

This subsection presents a series of graphs on the impact of the extent of the bailout, as parameterized by the parameter ϕ. The qualitative features of these graphs are exactly the same as those of the previous subsection, which varies p_d. Here, in Figure A6, the regulator sets capital requirements low when the extent of the bailouts, ϕ, is low, but sets higher capital requirements when ϕ rises above some critical level. As shown in Figure A7, Panels A through E, in the region of low capital requirements, optimal subordination and credit lines is zero; deposit funding is high; expected bank profits and government bailouts increase rapidly with ϕ; and the regulator's objective falls rapidly with ϕ. In the region of high capital requirements, subordination and credit lines are positive; deposit funding is low; expected bank profits and government bailouts increase, though not as rapidly, with ϕ; and the regulator's objective falls, although not as rapidly, with ϕ.

4 Conclusion

In this paper, we provide a conceptual framework to explain the observed transformation of risks between banks and NBFIs, where they coexist in a symbiotic relationship: credit risk migrates to NBFIs, but liquidity risk stays with banks, i.e., NBFIs provide equity subordination, but banks provide funding to NBFIs both in the form of senior loans and contingent credit lines.

We highlight the following features of our conceptual framework. First, our framework clarifies that subordination to NBFIs can be seen as a manifestation or mirror image of deposit-seeking by banks. As such, the advantages of deposit funding, e.g., convenience yield on deposits and access to safety nets, increase the bank-NBFI transformation whereby credit risks migrate to NBFIs and away from banks. Second, there is a trade-off in setting the level of capital requirements: higher capital requirements can reduce the costs of bailing out depositors, but can also increase inefficiencies from the regulatory arbitrage of increased subordination. Third, contingent provision of liquidity by banks to NBFIs, to mitigate the inefficiency of regulatory arbitrage in the form of subordination, can result in a transfer of credit risk back to banks. Hence, on-balance-sheet subordination to NBFIs may not reflect true or effective subordination and bank credit lines.

Several aspects of our framework can be extended further, investigated empirically, and considered in policy debates, so as to understand and capture more thoroughly the richness of bank-NBFI risk transformations.

One, the deposit funding advantages of banks have been shown to vary over the interest-rate cycle. Our framework therefore implies a corresponding cyclical variation between subordination to NBFIs and the interest-rate cycle. In particular, not only would the NBFI sector grow in times of low interest rates when banks enjoy a greater convenience yield on deposits, but this growth would be on the back of greater availability of bank credit lines. All of this assumes that bank capital requirements do not have a coincident variation, which is an assumption worth verifying.

Two, research has also shown that drawdowns of bank credit lines incur an encumbrance cost in the form of higher capital requirements for banks on drawn balances relative to undrawn balances. Our model did not feature an interim period between the drawdown of credit lines by NBFIs and the loan payoffs, that is, the materialization of credit risk. This feature, incorporated in a model extension, would imply an empirical regularity that loan losses do not have to materialize for NBFI stress to get transferred to bank balance sheets and to begin having implications for the real economy.

Three, our model implies that – because subordination arises due to coarseness of capital requirements in their sensitivity to NBFI subordination and access to bank credit lines – attempts at improving upon this sensitivity, while difficult to undertake in practice, might offer some low-hanging fruits. Considering when and how different categories of NBFIs draw down on bank credit lines, together with evaluating the consequences for bank capital adequacy, is a natural recommendation for bank stress tests. In short, bank-NBFI risk transformations might in some cases have first order implications for system-wide financial regulation.

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Appendix A Proofs

Proof to Proposition 1. When $\phi = 0$, the expected profits for zero subordination ($s = 0$) and a positive optimal subordination ($s > 0$ such that $w_s(s, 0) + 1 = 0$) are:

$$\pi(0,0) = p_u R_u + p_d R_d - (\beta r_f - y) - w(0,0)\left[r_f(1-\beta) + y\right],$$

$$\pi(s,0) = p_u(1-p^l)R_u + (p_d + p_u p^l)R_d - (\beta r_f - y) - \big(s + w(s,0)\big)\left[r_f(1-\beta) + y\right].$$

The difference in expected profits is:

$$\pi(0,0) - \pi(s,0) = p_u p^l (R_u - R_d) + \big(s + w(s,0) - w(0,0)\big)\left[r_f(1-\beta) + y\right].$$

This difference is strictly increasing in p^l. Since $(\pi(0,0) - \pi(s,0))|_{p^l=0} < 0$ and $(\pi(0,0) - \pi(s,0))|_{p^l=1} > 0$, the Intermediate Value Theorem ensures there exists a unique $\bar{p}^l \in (0,1)$ such that $(\pi(0,0) - \pi(s,0))|_{p^l=\bar{p}^l} = 0$. Thus, for $p^l \leq \bar{p}^l$, $\hat{s} > 0$.

When $\phi > 0$, the profit difference becomes:

$$\pi(0,0) - \pi(s,0) = p_u p^l (R_u - R_d) + \big(s + w(s,0) - w(0,0)\big)\left[p_d\phi + r_f(1-\beta) + y\right].$$

This expression remains strictly increasing in p^l. Evaluating it at $\bar{p}^l$ yields $(\pi(0,0) - \pi(s,0))|_{p^l=\bar{p}^l} = p_d\phi\big(s + w(s,0) - w(0,0)\big) < 0$. Therefore, there must exist a higher threshold $\bar{\bar{p}}^l > \bar{p}^l$ such that $(\pi(0,0) - \pi(s,0))|_{p^l=\bar{\bar{p}}^l} = 0$, implying $\hat{s} > 0$ for any $p^l \leq \bar{\bar{p}}^l$. ■

Proof to Proposition 2. When $\pi(0,0) > \pi(s,0)$, we have $\hat{s} = 0$.

When $\pi(s,0) > \pi(0,0)$, the first-order condition with respect to s is given by:

$$w_s(s,0) + 1 = 0.$$

Solving this condition yields $\hat{s} = \frac{1}{a}\ln(aw_0)$. This explicit expression for $\hat{s}$ does not depend on ϕ, which governs the bailout policy, and it is strictly increasing in w_0. ■

Proof to Proposition 3. When $p^l < \bar{p}^l$, we have

$$\pi(s_2, s_2) - \pi(s_1, 0) = p_u p^l (R_u - R_d) - (s_2 + w(s_2, s_2) - w(s_1, 0) - s_1)[r_f(1-\beta) + y],$$

where s_1 satisfies $w_s(s,0)+1 = 0$ and s_2 satisfies $\frac{dw(s,s)}{ds}+1 = 0$, i.e., s_1 is the optimal subordination in the case of *Subordination without Credit Lines* and s_2 is the optimal subordination in the case of *Subordination with Credit Lines*.

From Proposition 2, we have $s_1 = \frac{1}{a}\ln(aw_0)$. Given $w_0 > \frac{1}{a-e^{-a}}$, it follows that $0 < s_2 = \frac{1}{a}\ln\left(\frac{aw_0}{1+w_0e^{-a}}\right) < s_1$. Consequently,

$$s_2+w(s_2,s_2)-w(s_1,0)-s_1 = w_0[e^{-as_2}-(1-s_2)e^{-a}]-w_0[e^{-as_1}-e^{-a}] = w_0e^{-as_2}-w_0e^{-as_1}+w_0s_2e^{-a} > 0.$$

Note that $\pi(s_2,s_2)-\pi(s_1,0)$ is increasing in p^l, and at $p^l = 0$, we have $\pi(s_2,s_2) - \pi(s_1,0)|_{p^l=0} < 0$. When $p^l = \bar{p}^l$, $\pi(0,0) = \pi(s_1,0)$. However, if the bank were to offer the credit line, it would choose s_2 rather than 0, since $(0,0)$ is also in the choice set. Therefore, $\pi(s_2,s_2) > \pi(0,0)$. As a result, $\pi(s_2,s_2) - \pi(s_1,0)|_{p^l=\bar{p}^l} > 0$.

By the intermediate value theorem, there exists a $\bar{p}^{lc} < \bar{p}^l$, such that $\pi(s_2,s_2)-\pi(s_1,0)|_{p^l=\bar{p}^{lc}} = 0$.

Similarly, when $p^l > \bar{p}^l$, we establish that $\pi(s_2,s_2) > \pi(0,0) > \pi(s_1,0)$, which implies $\hat{L} > 0$. ■

Proof to Proposition 4. Suppose that the bank offers a credit line equal to the amount of subordination. In this case, the first-order condition with respect to s is given by:

$$\left(\frac{dw(s,s)}{ds}+1\right)[\phi(p_{db}+p_{dn})+r_f(1-\beta)+y]+p_{db}\phi = 0.$$

Solving this condition yields:

$$\hat{s} = \hat{L} = -\frac{1}{a}\ln\left(\frac{e^{-a}}{a}+\frac{A}{aw_0}\right),$$

where $A = \frac{\phi p_{dn}+r_f(1-\beta)+y}{\phi(p_{db}+p_{dn})+r_f(1-\beta)+y}$. Note that this expression is strictly increasing in w_0.

Therefore, given $p^l > \bar{p}^{lc}$, if w_0 is relatively small—specifically, $w_0 \le \frac{A}{a-e^{-a}}$—we have $\hat{L} = 0$, regardless of whether the bank subordinates. Conversely, when $w_0 > \frac{A}{a-e^{-a}}$, we have $\hat{L} = -\frac{1}{a}\ln\left(\frac{e^{-a}}{a}+\frac{A}{aw_0}\right)$, which is strictly increasing in w_0.

Consequently, the optimal credit line $\hat{L}$ is nondecreasing in w_0 overall. ■

Proof to Proposition 5. To analyze the regulator's choice, we first derive the first-order derivative of the regulator's optimization problem with respect to w_0:

$$\frac{d\pi^R(\hat{L},\hat{s})}{dw_0} = \left[\phi(p_{db}+p_{dn})(1-\delta) - \delta(r_f(1-\beta)+y)\right]\frac{\partial w}{\partial w_0} + \left[\phi(p_{db}+p_{dn})\frac{\partial w}{\partial \hat{s}} + \phi p_{dn} + \left(1-\frac{\partial \hat{L}}{\partial \hat{s}}\right)\phi p_{db}\right]\frac{d\hat{s}}{dw_0}.$$

Under the condition $p^l > \bar{p}^{lc}$, the bank optimally chooses a positive credit line $\hat{L} > 0$. In this regime, the relationship between subordination and the credit line is given by $\hat{s} = \hat{L}$, which implies $\frac{\partial \hat{L}}{\partial \hat{s}} = 1$. Consequently, the first-order derivative simplifies to:

$$\frac{d\pi^R(\hat{L},\hat{s})}{dw_0} = \left[\phi(p_{db}+p_{dn})(1-\delta) - \delta(r_f(1-\beta)+y)\right]\frac{\partial w}{\partial w_0} + \left[\phi(p_{db}+p_{dn})\frac{\partial w}{\partial \hat{s}} + \phi p_{dn}\right]\frac{d\hat{s}}{dw_0}.$$

To evaluate the concavity of the objective function, we take the second-order derivative with respect to w_0 when $\hat{L} > 0$:

$$\begin{aligned}\frac{d^2\pi^R(\hat{L},\hat{s})}{dw_0^2} &= \left[\phi(p_{db}+p_{dn})(1-\delta) - \delta(r_f(1-\beta)+y)\right]\frac{\partial^2 w}{\partial w_0^2} \\ &\quad + \left[\phi(p_{db}+p_{dn})\frac{\partial w}{\partial \hat{s}} + \phi p_{dn}\right]\frac{d^2\hat{s}}{dw_0^2}. \\ &= \left[\phi(p_{db}+p_{dn})(1-\delta) - \delta(r_f(1-\beta)+y)\right]\frac{\partial^2 w}{\partial w_0^2} \\ &\quad + \left[\phi(p_{db}+p_{dn})(-A) + \phi p_{dn}\right]\frac{d^2\hat{s}}{dw_0^2}.\end{aligned}$$

We know that $\frac{\partial w}{\partial w_0} = e^{-a\hat{s}} - (1-\hat{s})e^{-a}$, and therefore $\frac{\partial^2 w}{\partial w_0^2} = \left(-ae^{-a\hat{s}} + e^{-a}\right)\frac{d\hat{s}}{dw_0}$. From Proposition 4,$\hat{s} = \hat{L} = -\frac{1}{a}\ln\left(\frac{e^{-a}}{a} + \frac{A}{aw_0}\right)$. Substituting this expression yields $-ae^{-a\hat{s}} + e^{-a} = -\frac{A}{w_0}$, so that $\frac{\partial^2 w}{\partial w_0^2} = -\frac{A}{w_0}\frac{d\hat{s}}{dw_0}$.

Furthermore, $\frac{d\hat{s}}{dw_0} = \frac{A}{ae^{-a}w_0^2 + aAw_0}$, and hence $\frac{d^2\hat{s}}{dw_0^2} = -\frac{(2ae^{-a}w_0 + aA)A}{(ae^{-a}w_0^2 + aAw_0)^2}$. Equivalently, $\frac{d^2\hat{s}}{dw_0^2} = -\frac{A}{w_0}\frac{\left(\frac{2e^{-a}w_0}{A}+1\right)}{e^{-a}w_0 + A}\frac{d\hat{s}}{dw_0}$.

Substituting the expressions for the derivatives of w and $\hat{s}$, the second-order derivative can be written as:

$$-\frac{A}{w_0}\left[\phi(p_{db}+p_{dn})(1-\delta) - \delta(r_f(1-\beta)+y) + \phi p_{dn} - A\phi(p_{db}+p_{dn})\frac{\frac{2e^{-a}w_0}{A}+1}{\frac{e^{-a}w_0}{A}+A}\right]\frac{d\hat{s}}{dw_0}.$$

If the following inequality holds,

$$\delta < \frac{2e^{-a}+A}{Ae^{-a}+A^2}\frac{\phi p_{dn}}{\phi(p_{db}+p_{dn})+r_f(1-\beta)+y} - \left(\frac{2e^{-a}+A}{e^{-a}+A}-1\right)\frac{\phi(p_{db}+p_{dn})}{\phi(p_{db}+p_{dn})+r_f(1-\beta)+y}$$

then the second-order derivative is negative, implying the first-order derivative is decreasing in w_0. Denote the RHS as δ^1.

If additionally,

$$\delta < \frac{\phi(p_{db}+p_{dn})}{\phi(p_{db}+p_{dn})+r_f(1-\beta)+y}$$
$$- \frac{\phi p_{db}(r_f(1-\beta)+y)}{\phi(p_{db}+p_{dn})+r_f(1-\beta)+y}\frac{A}{(e^{-a}+A)(e^{-a}+A-ae^{-a}+e^{-a}\ln a - e^{-a}\ln(e^{-a}+A))},$$

then the first-order derivative is positive when $w_0 = 1$. Denote the RHS as δ^2.

Therefore, denote $\bar{\bar{\delta}} = \min\{\delta^1, \delta^2\}$. Since $p^l > \bar{p}^{lc}$, we know from Proposition 3 and Proposition 4 that $\hat{L} > 0$ when $w_0 = 1$.

Therefore, we can conclude that $w_0^* = 1$ when $\delta < \bar{\bar{\delta}}$. When $\hat{L} = 0$, $\frac{d\pi^R(\hat{L},\hat{s})}{dw_0} < 0$ when $\delta > \frac{\phi_b P_{db}+\phi_n P_{dn}}{\phi_b P_{db}+\phi_n P_{dn}+r_f(1-\beta)+y}$. Denote $\bar{\delta} = \frac{\phi_b P_{db}+\phi_n P_{dn}}{\phi_b P_{db}+\phi_n P_{dn}+r_f(1-\beta)+y}$.

■

Appendix B Figures

w/o Credit Line or w/ Undrawn Credit Line

Before Loan Realization

Bank		NBFI	
Assets	Liabilities	Assets	Liabilities
Sr. Tranche: $1\text{-}s$	Deposits: $1-s-w(s,L)$	Loan: 1	Sr. Tranche: $1\text{-}s$
	Equity: $w(s,L)$		Equity: s

Loan Default, Wiping Out NBFI Equity

Bank		NBFI	
Assets	Liabilities	Assets	Liabilities
Sr. Tranche: $1+R_d$	Deposits: $1-s-w(s,L)$	Loan: $1+R_d$	Sr. Tranche: $1+R_d$
	Equity: $\boldsymbol{R_d+s+w(s,L)}$		Equity: *0*

w/ Drawn Credit Line $L \leq s$ to Support Borrowing Firm

Before Liquidity and Loan Realizations

Bank		NBFI	
Assets	Liabilities	Assets	Liabilities
Sr. Tranche: $1\text{-}s$	Deposits: $1-s-w(s,L)$	Loan: $1+L$	Sr. Tranche: $1\text{-}s$
Credit Line: L	Fed Loan: L		Credit Line: L
	Equity: $w(s,L)$		Equity: s

NBFI Cannot Raise New Equity; Defaults on Credit Line; and Loan Defaults

Bank		NBFI	
Assets	Liabilities	Assets	Liabilities
Sr. Tranche: $1+R_d$	Deposits: $1-s-w(s,L)$	Loan: $1+R_d$	Sr. Tranche: $1+R_d$
Credit Line: *0*	Fed Loan: L		Credit Line: *0*
	Equity: $\boldsymbol{R_d+s-L+w(s,L)}$		Equity: *0*

Figure A1: NBFI Originates Loan

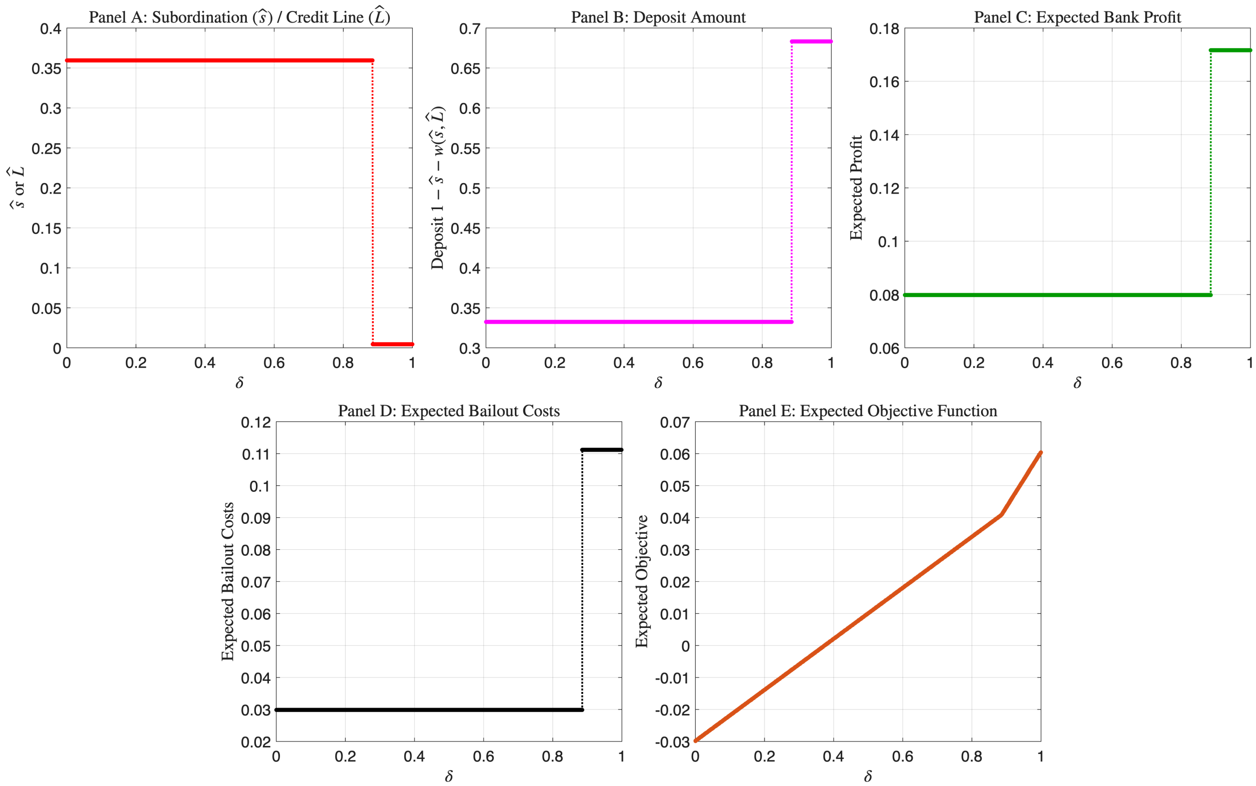

Figure A2: Impact of the Regulator's Weight on Equilibrium Variables

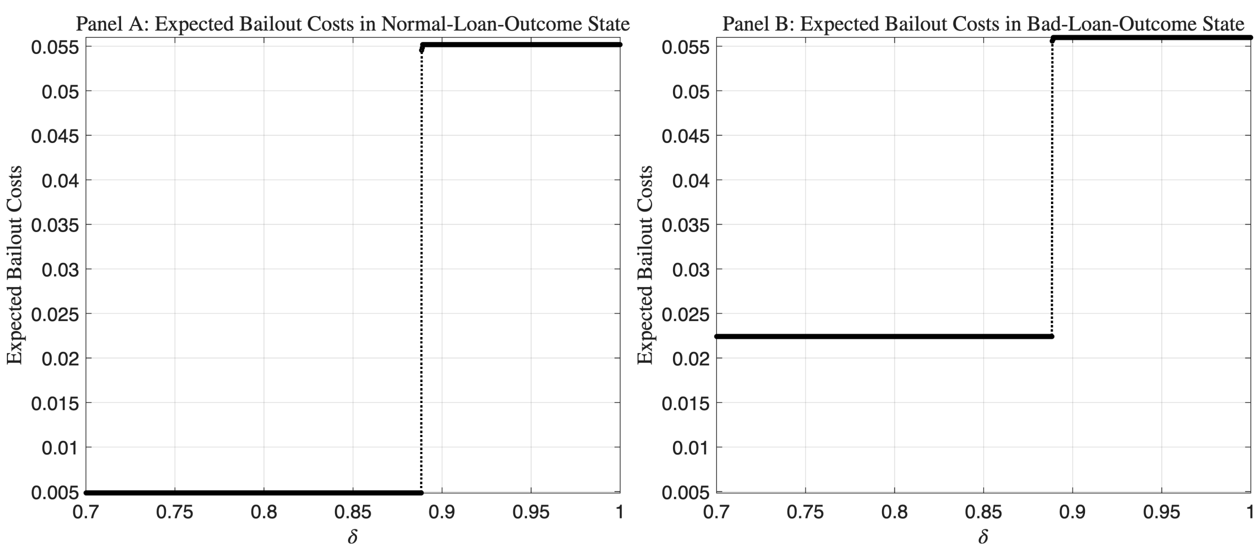

Figure A3: Expected Bailout Costs in Different Loan Outcome Pairs with Different Regulator's Weights

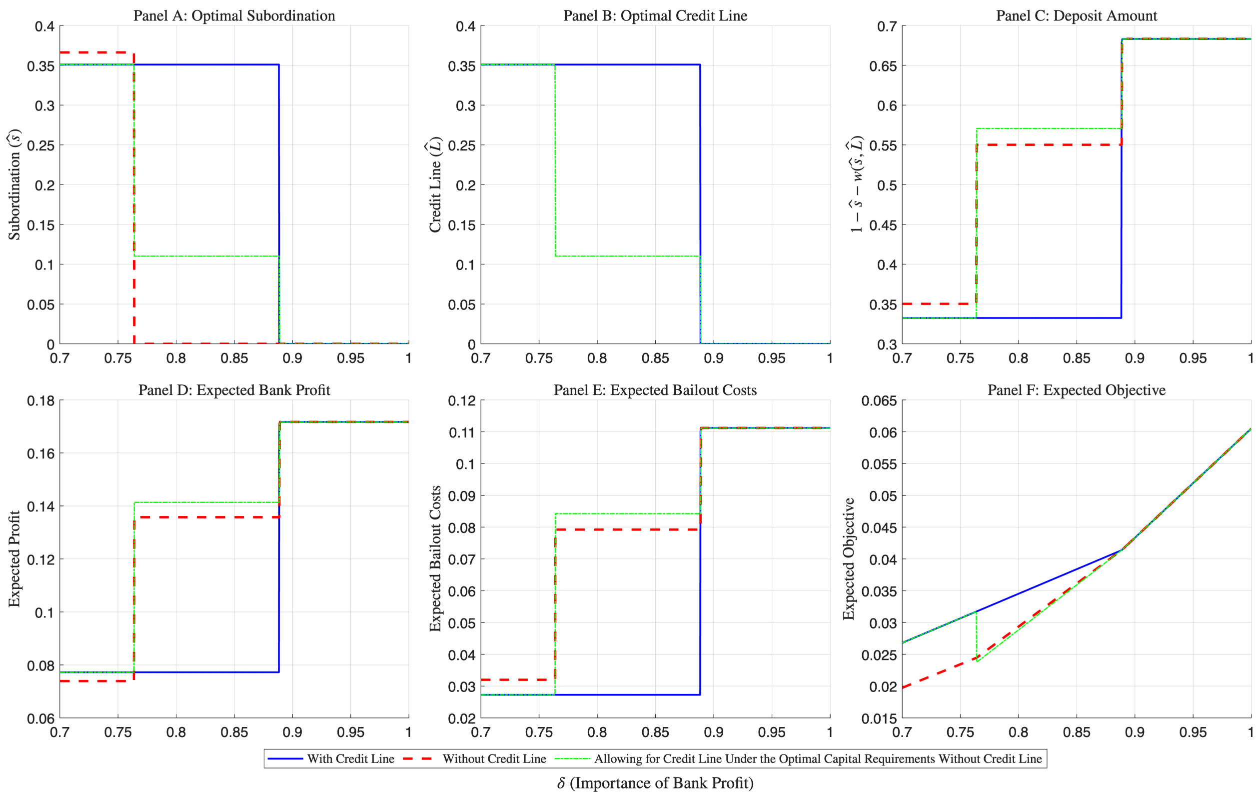

Figure A4: Impact of the Regulator's Weight on Equilibrium Variables with and without Credit-Line Provision

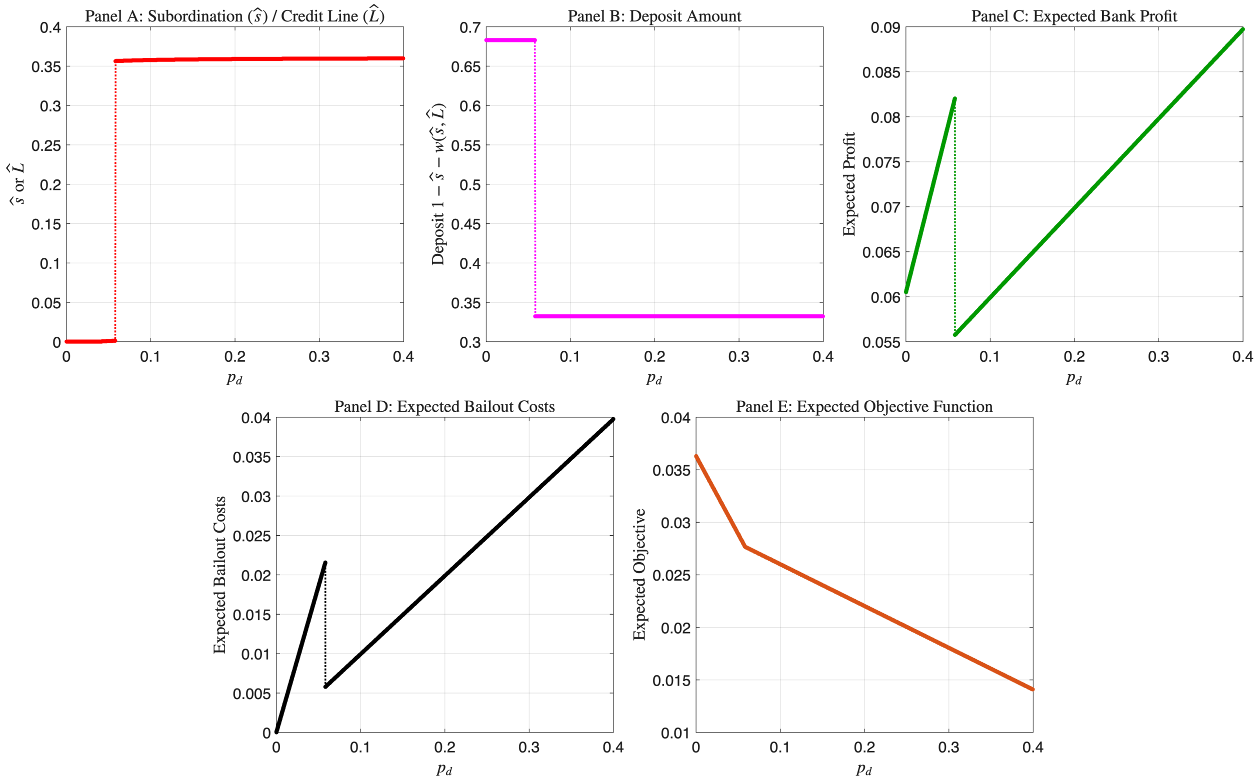

Figure A5: Impact of the Probability of Bad States on Equilibrium Variables

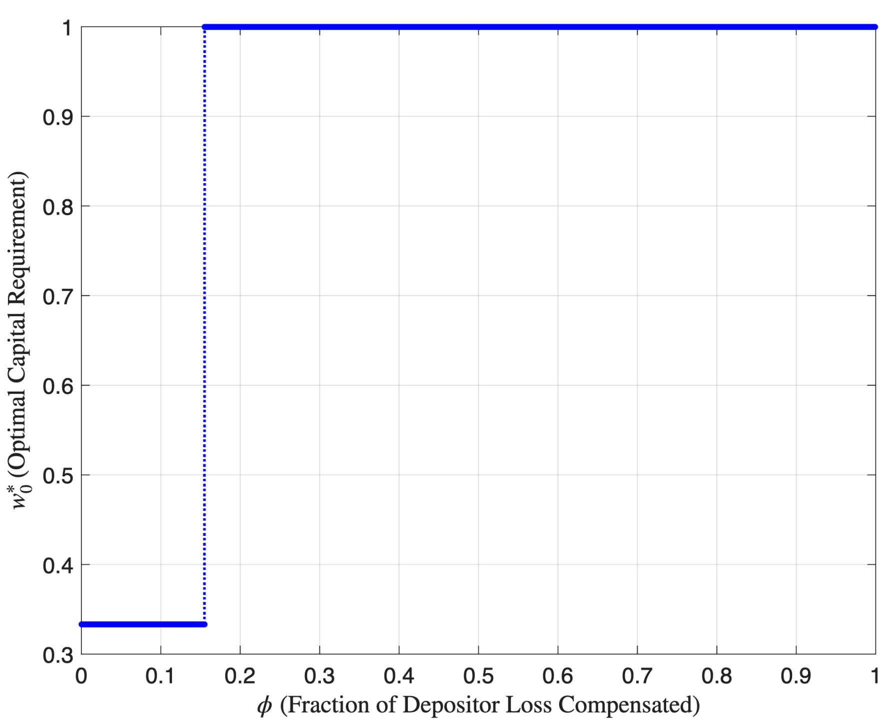

Figure A6: Optimal Capital Requirement as a Function of the Extent of the Bailout

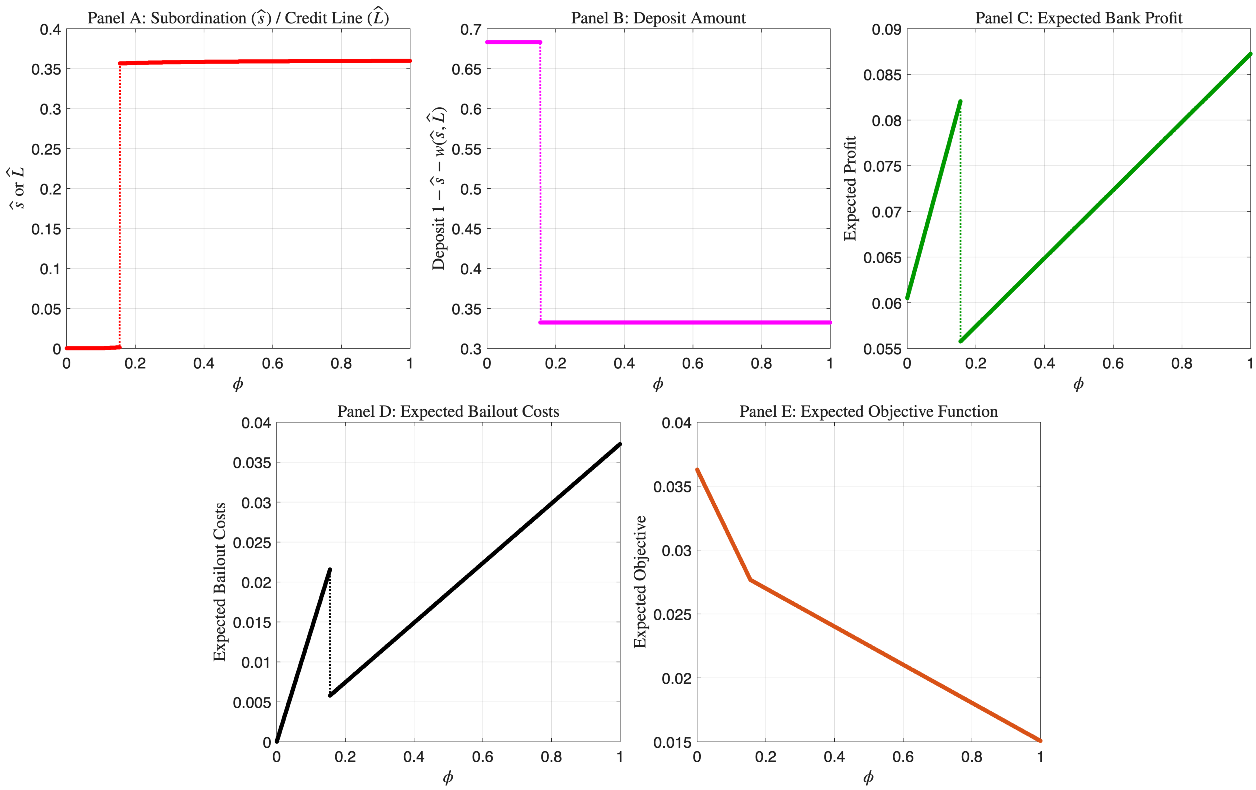

Figure A7: Impact of the Extent of the Bailout on Equilibrium Variables